# Trust Building in Credence Goods Markets 

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#### Abstract

We study trust building in credence goods markets in a dynamic setting. An extreme lemon problem arises in the one-shot game and results in no trade. In the repeated game, an expert's honesty is monitored through consumers' rejection of his recommendations. We characterize the most profitable equilibrium for any discount factor. The expert's maximal profit weakly increases in the discount factor but cannot achieve the first best because the most profitable equilibrium either involves insufficient treatment for the serious problem or excessive treatment for the minor problem. The monitoring technology and the equilibrium outcome contrast sharply with their counterparts for experience goods markets. Competition enhances efficiency by allowing consumers to use second opinions to monitor expert honesty more cost effectively, but the efficiency gain comes at the cost of less honesty.


Keywords: credence goods, trust, repeated game, efficiency, honesty, monopoly, competition, search JEL: D00, D42, D80, D82, D83, L00
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## 1 Introduction

In markets for professional services such as health care, consulting, taxi rides, and various repair services, buyers often lack expertise to identify the appropriate treatments or solutions for their problems and hence rely solely on experts for provision of both diagnosis and treatment/solution. Information asymmetry may prevail even after consumption. For instance, consider that you have a knee injury and your doctor recommends an arthroscopic knee surgery. You can verify whether the pain is relieved after the surgery, but it is hard to tell whether it could have been solved simply by changing your lifestyle. Goods and services with these features are termed "credence goods" by Darby and Karni (1973).

Credence-good sellers' expertise in diagnosing buyers' problems makes them "experts", but also provides them with opportunities to exploit their consumers. There is a great deal of documented evidence to demonstrate that expert fraud is very common and costly. CNN reported that Dr. Farid Fata in Michigan made millions of dollars from prescribing cancer treatment drugs to more than 500 patients who did not need them. ${ }^{1}$ Many non-cancer patients received expensive and aggressive treatments but only learned that they never needed those treatments after Dr. Fata's arrest in 2013. In a field experiment, Schneider (2012) found widespread under- and over-treatment in the auto repair market. Blafoutas et al. (2013) and Liu et al. (2017) find empirical evidence that taxi drivers take nonlocals for unnecessary detours.

When consumers are concerned about expert fraud, trust plays an important role in facilitating trade. It has been documented that patient compliance rates are nearly three times higher in primary care relationships characterized by very high levels of trust (See Atreja, et al. (2005) and Piette, et al. (2005)). Similarly, homeowners and car owners are more likely to follow recommendations made by handymen and mechanics whom they trust. Even though that trust plays an important role in guiding buyers' decisions when seeking professional services, the mechanism through which a credence-good seller builds trust in a repeated game has not been thoroughly studied in the literature and is not well understood.

In this paper, we adopt the notion of trust in the frontier repeated-game literature (see e.g.,Cabral, 2005 for definition of trust and how it differs from reputation). More specifically, in the context of credence-good

[^1]markets, consumer trust is defined as consumers' belief of the expert's level of honesty in reporting the nature of their problems. The fact that trust building for credence-good sellers in a dynamic setting has not received the attention it deserves may be driven by the presumption that trust building mechanism for credence goods is similar to the well-researched trust building mechanism for experience goods. ${ }^{2}$ Nevertheless, we argue that they are quite different. Experience-good buyers can monitor the quality of the good based on their consumption experiences. However, such monitoring technology is ineffective for credence goods because consumption and monitoring for honesty go against each other. Once buyers have received the services and their problems have been fixed, they cannot tell whether they really needed the service, i.e., whether the expert was honest.

We ask the following research questions: 1) How does a credence-good seller gain consumer trust in a repeated-game setting? 2) How do the seller's concerns for future business shape his conduct? 3) Is the efficient outcome attainable when the expert is sufficiently patient? 4) How does competition influence the trust building mechanism, market efficiency, and expert honesty?

In our model, a long-lived expert interacts with a sequence of short-lived consumers, each with a problem causing either a substantial or a minor loss. We thereafter call the problem serious or minor, respectively. Consumers do not know the nature of their problems and rely on the expert for diagnosis and treatment. The expert has one treatment which can fix both types of problem. It is efficient to use the treatment for the serious problem but inefficient to use it for the minor problem. This assumption allows us to analyze both over- and under-treatment.

In each period, the expert posts a price for his treatment. A consumer arrives in the market. She observes the price and decides whether to consult the expert for treatment. After diagnosis, the expert learns the nature of the consumer's problem. Then, he either recommends the treatment at the quoted price or recommends no treatment. If the expert recommends the treatment, the consumer decides whether to accept it. Once the consumer accepts the treatment, the expert is liable for fixing the consumer's problem. At the end of the period, the expert's price, his recommendation, the consumer's acceptance decision as

[^2]well as her utility from accepting or rejecting the expert's recommendation become public information. In other words, the consumer shares her experience with others. This simplifying assumption is adopted for tractability. ${ }^{3}$ It will be clear that even with this simplifying assumption, monitoring is still imperfect.

We assume the expected loss caused by the problem is lower than the cost of fixing the serious problem. ${ }^{4}$ Under this assumption, an extreme "lemon problem" emerges in the static game. The expert cannot gain consumer trust when he has no concerns for future business and the market collapses. This no-trade equilibrium holds whether the services are credence goods or experience goods because consumers' post-consumption experiences do not play any role in the one-shot game. ${ }^{5}$

We characterize the set of perfect public equilibrium payoffs of the repeated game for any discount factor and focus on the most profitable equilibrium, which is referred to as the optimal equilibrium thereafter. Since the expert's recommendation history is publicly observable, players' strategies can depend on the entire recommendation history and hence the strategy space is huge. Nevertheless, the expert can implement the optimal equilibrium using simple stationary strategies. As the discount factor increases, the expert's equilibrium profit gradually increases but cannot attain the first best. For any discount factor less than one, the optimal equilibrium either involves overtreatment for the minor problem or undertreatment for the serious problem. This stands in sharp contrast with canonical models of experience goods markets wherein the first best is attained when the discount factor is greater than a threshold. ${ }^{6}$

When the expert is sufficiently patient, in the optimal equilibrium he makes honest recommendations and charges a price high enough to induce consumers to reject the treatment sometimes. Due to the credence nature of the service, consumers cannot detect expert cheating via consumption. A consumer instead learns whether the expert has lied through refusing to accept the treatment recommendation and experiencing the

[^3]loss from the problem. To give a concrete example, let's revisit the case of a knee injury. Suppose you do not take the arthroscopic surgery recommended by your doctor and instead try resting and exercise therapy at home. If your injury is minor, the pain will be relieved after some time. But if it is a serious tear in the knee's meniscus, your pain will get worse and you may experience blockage of motion frequently, which reveals that the doctor has been honest about your condition. ${ }^{7}$

If the expert is proved to be honest, he gains the trust from future consumers who continue to accept his recommendations with a positive probability less than one. Otherwise, the expert loses consumers' trust and all future business. We refer to this equilibrium as the monitoring-by-rejection equilibrium. The expert wants to induce some rejection from consumers because honesty can only be monitored by rejection, which helps the expert gain trust of future consumers and continue to charge a high price. Although consumer rejection keeps the expert honest, it also results in inefficient undertreatment for the serious problem. Nevertheless, the efficiency of the equilibrium gradually increases in the discount factor and converges to (but never achieves) the first best because a smaller and smaller (yet still positive) rejection rate is required to keep the expert honest when he is more and more patient. It is worth noting that the monitoring-by-rejection equilibrium is a form of imperfect monitoring because lying is detected with an endogenously determined probability less than one.

When the expert becomes less patient, it is too costly for him to induce monitoring for honesty because it requires a high rejection rate of recommended treatment for monitoring-by-rejection to be effective. In the optimal equilibrium, the expert posts a low price for his treatment and recommends the treatment for both types of problems. The expert makes a loss from efficiently treating the serious problem but gains from inefficiently fixing the minor problem. Consumers always accept the expert's recommendations. We refer to this equilibrium as the one-price-fix-all equilibrium. This equilibrium is characterized by a low level of trust because consumers expect the expert to always recommend the treatment and never honestly report the nature of their problems. Although the expert has an incentive to cherry-pick the minor problem to treat,

[^4]his refusal to treat the serious problem will trigger the punishment phase in which he loses future business. Since the expert gives up inducing monitoring for honesty, the minor problem is fixed with probability one in equilibrium, causing inefficiency from overtreatment. Consumers' failure to monitor expert honesty through consumption stands in sharp contrast with experience goods market in which consumption facilitates monitoring the quality of the goods.

Our main model shows that concerns for consumers' trust do not warrant efficient treatment in credencegoods monopoly market. It is natural to ask how competition affects trust building, market efficiency and honesty. We consider an extension of the monopoly model to a competitive market consisting of $n$ experts with free entry and exit. There exists a symmetric equilibrium in which consumers actively search for honest recommendations after incurring a search cost and use second opinions to monitor expert honesty. In this equilibrium, all experts lie with a small probability. Upon the first treatment recommendation, a consumer randomizes between accepting the recommendation and soliciting a second opinion. When the consumer samples multiple opinions, she can identify cheating experts through observing conflicting recommendations. If an expert is caught cheating, he loses all future consumers, exits the market, and is replaced by a new expert. This competitive equilibrium is sustainable when the discount factor is sufficiently high and the search cost is sufficiently low.

The competitive equilibrium has some interesting features. When search cost is low, competition can improve market efficiency through a nonprice channel. The efficiency gain in our model stems from the more efficient monitoring technology for honesty used by consumers in the competitive market. In the monopoly market, consumers keep the expert honest at the expense of burning the surplus from fixing the serious problem through rejection. By contrast, in the competitive market, consumers monitor expert honesty by soliciting second opinions. A skeptical consumer may reject the first treatment recommendation, but she accepts the second recommendation with probability one. So, the serious problem is repaired with probability one in equilibrium.

Second, the introduction of competition may induce more dishonest recommendations. Note that the competitive equilibrium involves small but pervasive cheating among experts, but the monopoly expert makes
honest recommendations when the discount factor is sufficiently high. So, the more efficient monitoring technology in the competitive market may come at the cost of less honesty. This is because some expert lying is necessary to induce consumers' search for second opinions in the presence of a search cost. If experts all make honest recommendations, a consumer will not pay the search cost to sample another opinion given that she will receive the same recommendation.

Third, our analysis shows that in the competitive market, reducing search cost can improve recommendation honesty and market efficiency whereas increase in the number of experts is less effective and could even impede trade because it is harder to sustain the search equilibria. This suggest that it is welfare enhencing to regulate entry into markets for expert services and to promote consumer search for second opinions by reducing search cost.

Most of the existing literature on credence goods focuses on a one-time transaction between experts and consumers and studies different mechanisms that discipline expert behavior. Contributions in this regard include Pitchik and Scotter (1987), Wolinsky (1993), Fong (2005), Liu (2011), Emons (1997, 2001), Alger and Salanie (2006), Dulleck and Kerschbamer (2009), Fong et al. (2014), Bester and Dahm (2017), Chen et al. (2017). Dulleck and Kerschbamer (2006) provides a comprehensive review of the early literature in one-shot games.

Despite the extensive studies of credence goods in one-shot games, the role of trust in a dynamic setting of credence good has not been as thoroughly investigated. The paper most closely related to ours is Wolinsky (1993) which investigates experts' reputation concerns in Section 5 as an extension. There are important differences between his analysis of reputation concerns and ours. First, Wolinsky (1993) rules out supergame consideration. He considers an overlapping generation model where consumers live two periods and monitor experts by their own past experiences. We allow a consumer to observe the entire public history and base her strategy on it. Second, in his analysis of reputation concerns, Wolinsky assumes that consumers delegate treatment decisions to experts and commit to accepting any bill presented to them. We relax this assumption and allow consumers to reject a treatment recommendation and use rejection and/or second opinions as monitoring technologies. Ely and Välimäki (2003) and Frankel and Schwarz (2009) investigate
a long-lived expert's recommendation strategy when facing a sequence of short-lived consumers. Similar as Wonlinsky's (1993) analysis of experts' reputation concerns, both studies assume that consumers commit to accepting the chosen expert's recommendations, so they can only use the expert's past recommendation history to monitor honesty.

Taylor (1995) considers a dynamic model with two-sided moral hazard. In Taylor (1995), experts have incentives to recommend unnecessary services to a healthy durable good whereas the owner of the good has incentives to shirk in maintenance efforts. Taylor (1995) shows that short-term contract may give rise to low maintenance efforts or strategic delay in checkups. When short-term contract is inefficient, long-term contract in the form of actuarially fair insurance may implement the first best. Our paper differs from Taylor (1995) in several aspects. First, we do not consider consumers' moral hazard problem and focus on experts' cheating incentives. Second, we analyze expert equilibrium behavior in both monopoly and competitive markets and highlight how competition affects honesty and efficiency while Taylor (1995) only considers the competitive market. Lastly, contrasting Taylor's (1995) setting where the good is either diseased or healthy, in our setting, consumers are always diseased and it is efficient to fix the serious problem but inefficient to fix the minor problem. So, an insurance contract which ensures to fix both types of problem would not result in the first because because there will be overtreatment for the minor problem.

Pesendorfer and Wolinksy (2003) study experts' incentives to make an unobservable and costly diagnosis effort for a consumer who searches for correct diagnosis in a competitive market. Experts in their model have no concerns for future business and they focus on experts' moral hazard in diagnosis, assuming away the possibility that experts lie about their diagnosis. We study a repeated game and focus on how experts' concerns for future business affect their incentives to honestly report their diagnosis.

An interesting recent paper by Hafner and Taylor (2017) studies interactions between a long-lived expert and a sequence of short-lived clients who need advice from the expert to decide how much to invest in their projects. In the Section of Reputational Contracts, Hafner and Taylor (2017) studies how reputation concern affects the expert's incentives to acquire information and provide honest advice to the clients. In their model, the outcome of a client's investment is a noisy signal about the quality of the expert's advice. At the end of
each period, a client submits a positive or a negative referral which becomes public information. Hafner and Taylor (2017) focuses on the equilibrium that maximize client surplus. Punishment arises on the equilibrium path when the expert's advice turns out to be wrong. Inefficiency arises in their model because a short-lived client ignores the externality of her investment decisions on others, which leads her to under invest following reports of good news and over invest following reports of bad news. We focus on the equilibrium which maximizes the long-lived expert's profit and the optimal monopoly equilibrium does not involve punishment on the equilibrium path. The inefficiency of the monitoring-by-rejection equilibrium occurs because it is necessary for the expert to induce some rejection from consumers in order to prove his honesty.

There is an emerging body of literature studying the impact of reputation on expert conduct in laboratory experiments. Dulleck et al. (2011) found that reputation has no significant influence on expert honesty and Mimra et al. (2016) found that price competition undermines reputation building. In the above experiments, consumers delegate treatment decisions to their chosen experts and monitor the experts only through their past recommendations as in Wolinsky (1993), Ely and Välimäki (2003), and Frankel and Schwarz (2009). Based on a different setting, our findings suggest that consumer rejection and search for second opinions are important channels for monitoring expert honesty and facilitating trust building. It would be interesting to test in laboratory our findings that when consumers can choose whether to accept or reject a treatment recommendation, i) in a monopoly setting, when the expert is more patient, he is more likely to charge a high price and make honest recommendations, and ii) for high discount factors and low search costs, competition and consumers' search for second opinions improve market efficiency at the expense of honesty.

Our paper is broadly related to the literature of reputation building in markets for experience goods including Klein and Leffler (1981), Hörner (2002) and Park (2005). For an excellent review of the voluminous literature on reputation and trust of experience goods sellers, please see Bar-Isaac and Tadelis (2008). In these papers, consumers learn the value of the goods through consumption. Our paper differs from these papers in that consumption of the service does not generate any information about sellers' honesty.

To the best of our knowledge, we are the first to systematically compare monitoring technologies and the equilibrium outcomes of credence expert services and experience expert services in a dynamic setting.

Our analysis reveals that an expert's concerns for future business impact the equilibrium outcome differently when the expert service is a credence good versus an experience good.

## 2 Model

Environment and Players A risk neutral, long-lived expert interacts with an infinite sequence of risk neutral, short-lived consumers. In each period $t \in\{1,2, \ldots, \infty\}$, one consumer arrives with a problem which is either minor or serious. Denote by $l_{s}$ the loss from the serious problem and $l_{m}$ the loss from the minor problem, with $0<l_{m}<l_{s}$. We refer to $l_{s}$ and $l_{m}$ as substantial and minor losses, respectively. It is common knowledge that the problem is serious with probability $\alpha \in(0,1)$. The expert can perfectly diagnose the consumer's problem at zero cost. There is one treatment available for the expert to fix both types of problems. It costs the expert $c$ to apply the treatment on the serious problem and $c-\varepsilon$, for some $\varepsilon>0$, to apply it on the minor problem. ${ }^{8}$ We assume $l_{m}<c-\varepsilon<c<l_{s}$, so it is socially efficient to fix the serious problem but inefficient to fix the minor problem. This assumption allows us to study overtreatment, i.e., provision of a treatment whose cost outweighs its benefit to consumers, which is well documented in health care, car repair and legal services markets.

For algebraic simplicity, we assume $E(l) \equiv \alpha l_{s}+(1-\alpha) l_{m}<c$, which implies $\alpha<\widehat{\alpha} \equiv \frac{c-l_{m}}{l_{s}-l_{m}}$. Under this condition, there is no trade in the static game. When this condition is violated, both types of problems will be fixed at the consumer's expected loss in the static game. We discuss the case $\alpha \geq \widehat{\alpha}$ in Section 6 . Following the literature ${ }^{9}$, we adopt the assumption the expert is liable for fixing the consumer's problem once the consumer has accepted his recommendation. ${ }^{10}$

[^5]Payoffs The expert maximizes his expected discounted sum of profit with discount factor $\delta \in(0,1)$. The expert's profit from treating the serious problem at price $p$ is $p-c$, and that from treating the minor problem is $p-c+\varepsilon$. Each consumer maximizes her expected payoff. A consumer's payoff is $u=-l_{i}$ if she has problem $i \in\{m, s\}$ and the problem is left untreated. The consumer's payoff is $u=-p$ if the problem is fixed at price $p$. Note that under the liability assumption, the consumer receives the same utility once she accepts the treatment. The consumer's utility depends on the nature of her problem only when the problem is left untreated and the loss of it is realized.

Information The consumer does not know the nature of her problem and has to rely solely on the expert for diagnosis and treatment. The expert learns whether the consumer's problem is serious or minor after diagnosis. The cost of treatment incurred by the expert is unobservable to the consumer. At the end of a period, the prices charged by the expert, his recommendation, the consumer's acceptance decision as well as her utility become public information.

Timeline We summarize our model by describing the timeline of events in each period $t=1,2, \ldots$.

Stage 1 The expert posts a price $p_{t}$ for his treatment.

Stage 2 A consumer arrives in the market. Nature draws the loss of her problem according to the prior distribution of problems.

Stage 3 The consumer observes the price and consults the expert who perfectly diagnoses her problem and either proposes to fix the problem at the quoted price or refuses to treat the consumer.

Stage 4 If the expert offers to fix the problem, the consumer decides whether or not to accept the offer.

Stage 5 The expert and consumers observe the realization of a public randomization device, denoted by
$x_{t} .{ }^{11}$

[^6]Strategy and Equilibrium Concept Denote by $R_{t} \in\left\{p_{t}, \emptyset\right\}$ the recommendation made by the expert in Stage 3, where $p_{t}$ denotes a recommendation of treatment and $\emptyset$ denotes refusal to treat the consumer. The expert's recommendation policy is $\beta_{i t}, i=m, s$, where $\beta_{i t}$ denotes the probability that the expert recommends $p_{t}$ for problem $i$ and $1-\beta_{i t}$ is the probability that the expert recommends no treatment for problem $i$. Denote by $a_{t} \in\{0,1\}$ the consumer's acceptance decision, where 0 denotes rejection and 1 denotes acceptance. Let $\gamma_{t} \in[0,1]$ denote the probability that the consumer accepts price $p_{t}$. Formally, we denote $h_{t}=\left\{p_{t}, R_{t}, a_{t}, u_{t}, x_{t}\right\}$ as the public events that happen in period $t$ and $h^{t}=\left\{h_{n}\right\}_{n=1}^{t-1}$ as a public history path at the beginning of the period, with $h^{1}=\varnothing$. Let $H^{t}=\left\{h^{t}\right\}$ be the set of public history paths till time $t$. A public strategy for the expert is a sequence of functions $\left\{P_{t}, \beta_{m t}, \beta_{s t}\right\}_{t=1}^{\infty}$, where $P_{t}: H^{t} \rightarrow \mathbb{R}_{+}$and $\left(\beta_{m t}, \beta_{s t}\right): H^{t} \cup \mathbb{R}_{+} \cup\{m, s\} \rightarrow[0,1]^{2}$. The public strategy of the consumer is $\gamma_{t}: H^{t} \cup p_{t} \rightarrow[0,1]$.

Equilibrium Concept We focus on Perfect Public Equilibria (Henceforth PPE) in which the expert and consumers use public strategies and the strategies constitute a Nash equilibrium following every public history. It is without loss of generality to restrict attention to public strategies. In our game the expert has private information about consumers' problems whereas consumers do not have any private information. In repeated-game terminology, it is a game with a product monitoring structure. Mailath and Samuelson (2006) have shown that every sequential equilibrium outcome is a perfect public equilibrium outcome in this case; so, there is no need to consider private strategies.

## 3 Static game

We first show that market collapses when the expert has no concerns for future business.

Lemma 1 There is no trade in the static equilibrium.

Given that it is efficient to fix the serious problem but inefficient to fix the minor problem, whenever trade happens, the serious problem must be repaired with a positive probability. For the expert to be willing to treat the serious problem, he must charge a price $p \geq c$. However, the consumer will reject such a price because once she accepts the price with a positive probability, the expert will recommend the treatment for both types of problems. This will yield the consumer a negative expected payoff given $E(l)<c \leq p$. The
assumption $E(l)<c$ imposes an upper bound on the likelihood of the serious problem. Under this condition, an extreme "lemons problem" develops. The expert cannot gain trust from consumers and the market for the expert's services completely shuts down, causing undertreatment of the serious problem.

We close this section by pointing out that the no-trade outcome continues to hold even if the expert's service is an experience good. To make this statement precise and also for later reference, we define experience expert service as follows:

Experience Service by Expert The expert's service is either of high match value or of low match value to the consumer. The probability of high match value is $\alpha \in(0,1)$, and the match value is the expert's private information prior to the consumer's consumption of the expert's service. When no treatment is provided, both players' payoffs are zero. When the match value is high, it costs the expert $c$ to serve the consumer and his service provides a benefit of $l_{s}>0$ to the consumer. When the match value is low, the cost to the expert and benefit to the consumer are respectively $c-\varepsilon$ and $l_{m}$. The consumer learns her match value after consumption because $l_{s} \neq l_{m}$.

When the expert provides experience services, the consumer learns the match value of the expert's service through her post consumption payoffs. It is easy to see that under the same assumptions on the parameter values, there is still no trade in equilibrium, because there are no more moves in the game after the consumer learns her actual problem, so whether the consumer learns her actual problem or not has no bearing on the players' earlier actions. In fact, the observation that there is no difference between the static equilibrium in monopoly market for credence goods and that for experience goods holds in general. We show in Section 6 that this statement remains true when $\alpha \geq \widehat{\alpha}$.

## 4 Repeated game

In this section, we study how the expert's conduct is affected by his concerns for future business. Because the expert is a monopolist who moves first in each period, we restrict attention to the equilibrium which yields the expert the highest profit. To illustrate how the monitoring technology for credence services differs from that for experience services and to explore its implication, we first discuss the optimal equilibrium in
experience services markets. The characterization of the optimal equilibrium in credence services markets follows.

### 4.1 Benchmark: experience service markets

We begin with the experience service markets as defined in the previous section and demonstrate that honesty and efficiency are jointly achieved when the discount factor is above a threshold. To see this, consider the following strategies: In each period, the expert posts price $l_{s}$ for the treatment and recommends the treatment only for the serious problem. Consumers accept the expert's recommendations as long as he did not recommend the treatment for the minor problem in the past. The game reverts to the static Nash equilibrium perpetually, otherwise. Given the expert's strategy, it is the consumer's best response to accept the treatment recommendation with probability one, which yields the expert an average profit of $\alpha\left(l_{s}-c\right)$. Since this is the surplus from the first best, it is the highest attainable profit for the expert.

Now, consider that the expert recommends the treatment for the minor problem. He receives a profit $l_{s}-c+\varepsilon$ in the current period but will lose all future business because his fraudulent recommendation will be detected with probability one and hence he will be punished from the next period onward. The no lying condition requires

$$
\underbrace{\frac{\delta}{1-\delta} \alpha\left(l_{s}-c\right)}_{\text {future loss }} \geq \underbrace{l_{s}-c+\varepsilon}_{\text {current gain }}
$$

which is satisfied if

$$
\begin{equation*}
\delta \geq \underline{\delta}^{e}(\alpha) \equiv \frac{l_{s}-c+\varepsilon}{l_{s}-c+\varepsilon+\alpha\left(l_{s}-c\right)} . \tag{1}
\end{equation*}
$$

So, for a given $\alpha$, the expert can achieve the first best profit by making honest recommendations when the discount factor is greater than the cutoff $\underline{\delta}^{e}(\alpha)$. This is a common property of experience good models with perfect monitoring, with some difference in details. Here, the current gain from cheating is the one-period profit from treating the consumer's minor problem while charging her for the benefit of treating the serious problem, whereas in a standard experience good setting, the current gain from cheating is the cost saving from producing the low-quality instead of the high-quality good.

### 4.2 Optimal equilibrium in credence goods markets

Now, we characterize the optimal equilibrium in markets for credence services. We show that for a given parameter configuration, the optimal equilibrium is either the so called "monitoring-by-rejection equilibrium" or the "one-price-fix-all equilibrium". We first characterize these two equilibria. Then, we prove that the expert's profit in any perfect public equilibrium is bounded above by his profit from one of these two equilibria. We then characterize the condition under which the monitoring-by-rejection equilibrium is more profitable than the one-price-fix-all equilibrium.

Monitoring-by-rejection equilibrium We begin by characterizing the monitoring-by-rejection equilibrium in the following proposition. Because the monitoring-by-rejection equilibrium is stationary, we suppress the subscript $t$.

Proposition 1 In the monitoring-by-rejection equilibrium, the expert posts $p=l_{s}$ for his treatment; he recommends the treatment for the serious problem and no treatment for the minor problem, i.e., $\left(\beta_{m}, \beta_{s}\right)=$ $(0,1)$. The consumer accepts the treatment recommendation at $p=l_{s}$ with probability $\gamma^{*}=1-\frac{1-\delta}{\delta \alpha} \frac{l_{s}-c+\varepsilon}{l_{s}-c}$ as long as in the past the expert was not caught i) recommending the treatment for the minor problem, or ii) refusing to fix the serious problem, or iii) deviating in price. Otherwise, the game perpetually reverts to the static Nash equilibrium in which there is no trade. The average profit of the monitoring-by-rejection equilibrium is

$$
\pi^{m} \equiv \alpha\left(l_{s}-c\right)-\frac{1-\delta}{\delta}\left(l_{s}-c+\varepsilon\right) .
$$

This equilibrium is sustainable for $\alpha \in(0, \widehat{\alpha})$ and $\delta \in\left[\underline{\delta}^{m}(\alpha), 1\right)$, with $\underline{\delta}^{m}(\alpha)=\underline{\delta}^{e}(\alpha) \equiv \frac{l_{s}-c+\varepsilon}{l_{s}-c+\varepsilon+\alpha\left(l_{s}-c\right)}$.

In the monitoring-by-rejection equilibrium, the expert makes honest recommendations and charges consumers their loss from the serious problem, which are similar to the experience-service seller's strategy in the optimal equilibrium. Nevertheless, consumers reject the recommendation with a positive probability, which results in undertreatment for the serious problem. As the expert becomes more patient, consumer acceptance rate gradually increases but never reaches full acceptance. As a result, the expert's profit as well as market efficiency gradually increase as the expert becomes more patient but never achieve the first best
levels. This stands in sharp contrast with experience goods markets in which honesty and full efficiency are jointly achieved when the seller's discount factor is above a cutoff value.

To understand this equilibrium, first note that although the expert makes honest recommendations on the equilibrium path, it is consumers' best response to sometimes reject the treatment offer because the treatment price is so high that they are just indifferent between accepting and rejecting the treatment recommendation.

To support the monitoring-by-rejection equilibrium, we have to consider both on-schedule and off-schedule deviations by the expert. The expert can make an off-schedule deviation to a price $p^{\prime} \neq l_{s}$. We assume that consumers believe the expert will recommend the treatment for both types of problems at $p^{\prime} \in\left[c, l_{s}\right)$ and for the minor problem at $p^{\prime} \in[c-\varepsilon, c)$. Given this off-equilibrium belief, consumers will reject $p^{\prime}$ with probability one, yielding the expert zero profit in the current period. In addition, a price deviation will trigger the punishment phase from next period onwards. So, there is no profitable price deviation.

The expert can also make an on-schedule deviation by recommending the treatment for the minor problem. Since consumers accept the treatment with a positive probability, the on-schedule deviation is not always detected and punished, so the equilibrium involves imperfect public monitoring. Consider that a consumer has the minor problem. If the expert recommends the treatment, he gains a profit $l_{s}-c+\varepsilon$ in the current period if his recommendation is accepted. Nevertheless, the expert risks losing all future business if his recommendation is rejected and he is caught lying. Consumers' acceptance rate $\gamma^{*}$ balances the trade off the expert is facing and makes him just indifferent between recommending and not recommending the treatment for the minor problem. Hence, it is the expert's best response to recommend no treatment for the minor problem. Specifically, the following no-lying condition holds at $\gamma^{*}$ :

$$
\begin{equation*}
\underbrace{\overbrace{\left(1-\gamma^{*}\right)}^{\text {Prob of being caught lying }} \times \frac{\delta}{1-\delta} \overbrace{\alpha\left(l_{s}-c\right) \gamma^{*}}^{\text {profit per period }}}_{\text {future loss }}=\underbrace{\gamma^{*}\left(l_{s}-c+\varepsilon\right)}_{\text {current gain from lying }} \tag{2}
\end{equation*}
$$

yielding a positive $\gamma^{*}=1-\frac{(1-\delta)\left(l_{s}-c+\varepsilon\right)}{\delta \alpha\left(l_{s}-c\right)}$ for $\delta>\underline{\delta}^{m}(\alpha)$. Because consumers' acceptance rate is less than one, the serious problem is sometimes left unrepaired, resulting in inefficiency from undertreatment.

Consumer acceptance rate $\gamma^{*}$ is less than one but increases in $\delta$ and $\alpha$. So, the efficiency of the equilibrium
increases when the expert is more patient or the consumer's problem is more likely to result in a substantial loss. If the expert cares more about future business, a smaller consumer rejection rate is sufficient to deter him from cheating. As the discount factor approaches one, $\gamma^{*}$ converges to (although never reaching) one and the monitoring-by-rejection equilibrium approaches (although never reaching) full efficiency. This suggests that it is less costly for the expert to gain consumer trust when he is more patient. Nevertheless, consumers cannot always accept the treatment because full acceptance of the treatment makes it impossible for consumers to monitor expert honesty. As a result, the expert will always recommend the treatment, causing the monitoring-by-rejection equilibrium to collapse. The acceptance rate $\gamma^{*}$ also increases in the likelihood of the serious problem $(\alpha)$. This is because the expert's equilibrium profit increases in $\alpha$, so he bears a larger future loss from lying when $\alpha$ increases and therefore a smaller rejection rate is sufficient to keep him honest.

It is worth noting that the minimum discount factor necessary to sustain the monitoring-by-rejection equilibrium coincides with that required to sustain the honest and efficient equilibrium in experience services markets. This is because the minimum discount factor $\underline{\delta}^{m}(\alpha)$ is derived when the equilibrium acceptance rate $\gamma^{*}$ is zero. Canceling out one $\gamma^{*}$ from both sides of the equation (2) and evaluating the simplified equality at $\gamma^{*}=0$, this condition coincides with the no lying condition (1) in experience goods markets when the equality holds. So, honesty can be achieved in both credence goods and experience goods markets when $\delta \geq \underline{\delta}^{m}(\alpha)=\underline{\delta}^{e}(\alpha)$, but the first best is not attainable in the former market.

Fong (2005) studies a one-shot game in which there are two treatments, and it is efficient to use the expensive treatment for the serious problem and the inexpensive treatment for the minor problem. In Fong's equilibrium, consumers reject the expensive treatment with a positive probability to keep the expert honest. Although Fong's equilibrium shares some similar features to the monitoring-by-rejection equilibrium, the driving forces for Fong's equilibrium and ours are completely different. Fong considers a short-lived expert, so honesty is driven by current profit consideration, not concerns for future business. In Fong's equilibrium, consumers reject the expensive treatment with a positive probability but accept the inexpensive treatment with probability one. So, when recommending the expensive treatment for the minor problem, the expert faces the trade-off between a higher profit margin and a lower acceptance rate, which balances off his cheating
incentives. In contrast, in our setting, because it is inefficient to fix the minor problem, the expert cannot make a positive profit from honestly reporting the minor problem in a static setting and hence will always misreport the minor problem as the serious problem, causing the market in the one-shot game to collapse. As a result of the different driving forces for expert honesty, the degree of the expert's patience plays a crucial role in our equilibrium, but it does not affect the equilibrium in Fong as the expert is short-lived. ${ }^{12}$

The following comparative statics are derived directly from Proposition 1 (proof omitted). These comparative statics are useful for comparing the profit of the monitoring-by-rejection equilibrium with that of the one-price-fix-all equilibrium which will be characterized at the end of this section.

Corollary $1 \pi^{m}$ strictly increases in $\alpha$ and $\delta$; the cutoff discount factor $\underline{\delta}^{m}(\alpha)$ strictly decreases in $\alpha$.

The expert's average profit increases in $\alpha$ and $\delta$ because consumers' acceptance rate $\gamma^{*}$ increases in these parameters. To see $\underline{\delta}^{m}(\alpha)$ decreases in $\alpha$, note that the expert's equilibrium profit is higher when the problem is more likely to be serious. So, the expert bears a larger future loss from lying and therefore is willing to make honest recommendations at a lower discount factor.

One-price-fix-all equilibrium We now turn to the one-price-fix-all equilibrium. In this equilibrium, the expert charges a consumer the average loss of her problem and always recommends the treatment. Consumers do not monitor expert honesty. Instead, they expect the expert to always repair their problems at the quoted price even though it is inefficient to repair the minor problem, and the expert's refusal to treat a consumer will trigger the punishment phase.

Define $\widetilde{\alpha} \equiv \frac{c-\varepsilon-l_{m}}{l_{s}-\varepsilon-l_{m}}$ and $\underline{\delta}^{o}(\alpha) \equiv \frac{c-E(l)}{(1-\alpha) \varepsilon}$. We characterize below the one-price-fix-all equilibrium which is also stationary:

Proposition 2 In the one-price-fix-all equilibrium, the expert posts $p=E(l)$ for the treatment and recommends the treatment irrespective of the consumer's problem. Consumers accept the expert's treatment with probability one as long as he recommended the treatment to all previous consumers. Otherwise, the game

[^7]reverts to the static Nash equilibrium perpetually. The average profit of the equilibrium is
$$
\pi^{o} \equiv \alpha\left(l_{s}-c\right)-(1-\alpha)\left(c-\varepsilon-l_{m}\right)
$$

For $\alpha \in(\widetilde{\alpha}, \widehat{\alpha})$, the one-price-fix-all equilibrium is sustainable if and only if $\delta \in\left[\underline{\delta}^{o}(\alpha), 1\right)$; it is not sustainable for $\alpha \in(0, \widetilde{\alpha}]$.

Since $l_{m}<E(l)<l_{s}$, consumers are overcharged for repairing the minor problem but receive a positive surplus from fixing the serious problem, which makes it optimal for them to accept the expert's recommendation with probability one. In this equilibrium, the expert is expected to always repair consumers' problems, so he does not have any on-schedule deviation. A price deviation or refusal to treat consumers are off-schedule deviations which are perfectly observed by consumers and will be punished from the next period onward. So, this equilibrium involves perfect public monitoring.

The condition $\widetilde{\alpha}<\alpha$ ensures that the equilibrium price $E(l)$ is greater than the average cost of fixing the problem so that the expert can earn a positive expected profit. Nevertheless, given the assumption $E(l)<c$, the price is too low to cover the expert's cost of repairing the serious problem. So, the expert has an incentive to cherry-pick the consumers with the minor problem and refuse to treat those with the serious problem. When the discount factor is greater than the cutoff $\underline{\delta}^{o}(\alpha)$, the expert's current gain from dumping a consumer with the serious problem is outweighed by his loss from losing all future business, and the equilibrium is sustainable.

In the one-price-fix-all equilibrium, consumers always consume the expert's services on the equilibrium path, and yet the expert does not honestly report the loss of their problems and the equilibrium involves inefficiency from overtreatment for the minor problem. This draws sharp contrast with experience goods markets wherein full consumption on the equilibrium path facilitates monitoring for quality and induces honest recommendations as well as efficient treatment provision.

Another set of comparative statics useful for comparing the monitoring-by-rejection equilibrium and one-price-fix-all equilibrium follows (proof omitted):

Corollary $2 \pi^{o}$ strictly increases in $\alpha$ and remains constant in $\delta$. When $\alpha \in(\widetilde{\alpha}, \widehat{\alpha})$, the cutoff discount
factor $\underline{\delta}^{o}(\alpha)$ strictly decreases in $\alpha$.

Unlike the monitoring-by-rejection equilibrium, the efficiency of the one-price-fix-all equilibrium does not increase in the expert's discount factor, so $\pi^{o}$ remains constant in $\delta$. In addition, $\pi^{o}$ increases in $\alpha$ because it is efficient to fix the serious problem but inefficient to fix the minor problem. It can be verified that the derivative $\frac{\partial \delta^{\circ}(\alpha)}{\partial \alpha}=-\frac{l_{s}-c}{(1-\alpha)^{2} \varepsilon}<0$. This is because when $\alpha$ is higher, the expert can charge a higher price for the treatment, so that he has a smaller gain from dumping costly consumers. In addition, because $\pi^{o}$ increases in $\alpha$, the expert has more future profits to lose from rejecting consumers as $\alpha$ increases. So, it is easier to sustain the one-price-fix-all equilibrium when the likelihood of the serious problem is higher.

In our game, because the expert's recommendation history is publicly observable, the expert and consumers can base their strategies on the entire recommendation history or some summary statistics such as frequency of the treatment recommendation. Nevertheless, the next proposition shows that it is without loss of generality to focus on the monitoring-by-rejection and the one-price-fix-all equilibria because the expert's maximum profit from any PPE is bounded above by the maximum profits from these two equilibria.

Proposition 3 For a given pair of $(\alpha, \delta)$, with $(\alpha, \delta) \in(0, \widehat{\alpha}) \otimes(0,1)$, the expert's average profit in any Perfect Public equilibrium is bounded above by $\max \left\{\pi^{m}, \pi^{o}\right\}$.

To prove Proposition 3, we adopt the approach pioneered by Abreu, Pearce and Stacchetti (1990) to characterize the perfect public equilibrium payoff set for the expert and show that the upper bound of the payoff set is either $\pi^{m}$ or $\pi^{o}$. There are five possible public outcomes in our game: 0) the expert recommends no treatment and the consumer suffers a minor loss, 1) the expert recommends no treatment and the consumer suffers a substantial loss, 2) the expert recommends the treatment, the consumer accepts the treatment and receives utility $-p, 3)$ the expert recommends the treatment, the consumer rejects it and suffers a minor loss, and 4) the expert recommends the treatment, the consumer rejects it and suffers a substantial loss. The set of public outcomes is denoted by $Y=\{0,1,2,3,4\}$.

An action profile $\sigma=\left(p, \beta_{m}, \beta_{s}, \gamma\right)$ determines the probability distribution over the public outcomes $f(y \mid \sigma), y \in Y$. The action profile $\sigma \in \boldsymbol{\Sigma}$ is enforceable on a payoff set $W \subseteq R^{2}$ if there exists a mapping $\nu$ :
$Y \rightarrow W$ such that for each player $i$ and $\sigma_{i}^{\prime} \in \Sigma_{i}$,

$$
\begin{aligned}
u_{i}(\sigma, \nu) & =(1-\delta) u_{i}(\sigma)+\delta \sum_{y \in Y} \nu(y) f(y \mid \sigma) \\
& \geq(1-\delta) u_{i}\left(\sigma_{i}^{\prime}, \sigma_{-i}\right)+\delta \sum_{y \in Y} \nu(y) f\left(y \mid \sigma_{i}^{\prime}, \sigma_{-i}\right)
\end{aligned}
$$

In other words, player $i$ 's action maximizes his/her payoff given other players' actions and the continuation payoff $\nu(y)$. The payoff set $W$ is self generating if for every element $w \in W$, there exists an action profile $\sigma \in \boldsymbol{\Sigma}$, enforced by $\nu$ on $W$, such that $w_{i}=u_{i}(\sigma, v)$. The set of perfect public equilibrium payoff is the maximum self-generating set. (See Chapters 7 and 8 in Mailath and Samulson 2006).

In our model, consumers are all short lived and therefore their continuation payoffs are zero. Therefore, we maximize the self-generating set $W_{i}, i$ being the expert, subject to short-lived consumers' incentive constraint

$$
\gamma=\underset{\gamma^{\prime}}{\arg \max } \gamma^{\prime}\left(E\left(l \mid \beta_{m}, \beta_{s}\right)-p\right)
$$

and we show that the upper bound of $W_{i}$ is $\max \left\{\pi^{m}, \pi^{o}\right\}$. The proof is relegated to Appendix B.

Proposition 3 is important because it allows us to focus on $\pi^{m}$ and $\pi^{o}$ to pin down the optimal equilibrium for any parameter configurations. Recall Proposition 2 that when $\alpha \leq \widetilde{\alpha}$, the one-price-fix-all equilibrium is not sustainable for any $\delta<1$ and hence $\pi^{m}$ is the highest attainable profit in this parameter range. For $\alpha \in(\widetilde{\alpha}, \widehat{\alpha})$, both the monitoring-by-rejection and the one-price-fix-all equilibria are sustainable when the discount factor is sufficiently high. In the following analysis, we characterize the condition under which the monitoring-by-rejection equilibrium dominates the one-price-fix-all equilibrium. To begin, we first compare the minimum discount factors necessary to support each type of equilibrium.

Lemma 2 There exists a unique cutoff likelihood $\bar{\alpha} \in(\widetilde{\alpha}, \widehat{\alpha})$ (See Figure 1) such that $0 \leq \underline{\delta}^{m}(\alpha) \leq \underline{\delta}^{o}(\alpha) \leq 1$ for $\alpha \in(\widetilde{\alpha}, \bar{\alpha}]$ and $0 \leq \underline{\delta}^{o}(\alpha)<\underline{\delta}^{m}(\alpha) \leq 1$ for $\alpha \in(\bar{\alpha}, \widehat{\alpha})$.

By Corollaries 1 and 2, the cutoff discount factors $\underline{\delta}^{m}(\alpha)$ and $\underline{\delta}^{o}(\alpha)$ are both decreasing in $\alpha$ and are illustrated in Figure 1. It is best to understand the comparison by considering the two extreme cases: $\alpha=$ $\widetilde{\alpha}$ and $\alpha=\widehat{\alpha}$. First, consider $\alpha=\widetilde{\alpha}$. Proposition 2 shows that the one-price-fix-for-all equilibrium is not sustainable for all $\delta<1$ at $\widetilde{\alpha}$. By contrast, the monitoring-by-rejection equilibrium is sustainable as long
as $\delta$ is sufficiently close to one, i.e., $\delta>\underline{\delta}^{m}(\widetilde{\alpha}) \in(0,1)$. By continuity, $\underline{\delta}^{m}(\alpha)$ lies below $\underline{\delta}^{o}(\alpha)$ when $\alpha$ is slightly greater than $\widetilde{\alpha}$. As $\alpha$ approaches to $\widehat{\alpha}$, it is easier to support the one-price-fix-for-all equilibrium. Note that in Figure 1, $\underline{\delta}^{o}(\widehat{\alpha})=0$. This is because when $\alpha=\widehat{\alpha}$, consumers' average loss $E(l)$ equals the treatment cost for the serious problem. Hence, it is optimal for the expert to repair the serious problem even when he has no concerns for future business. By contrast, in order to support the monitoring-by-rejection equilibrium, the expert's discount factor must be positive. If the expert has no concerns for future business, he would strictly prefer recommending the treatment for the minor problem because he has a positive gain in the current period from lying.

Both the monitoring-by-rejection and the one-price-fix-all equilibria are sustainable for $\alpha \in(\widetilde{\alpha}, \widehat{\alpha})$ and $\delta>\max \left\{\underline{\delta}^{o}(\alpha), \underline{\delta}^{m}(\alpha)\right\}$ (refer to Figure 1). In this case, the most profitable equilibrium depends on the comparison between $\pi^{m}$ and $\pi^{o}$. For a fixed $\alpha$, define $\delta_{1}(\alpha) \equiv \frac{l_{s}-c+\varepsilon}{l_{s}-c+\varepsilon+(1-\alpha)\left(c-\varepsilon-l_{m}\right)}$ which is solved from $\pi^{m}=\pi^{o}$. Next, we characterize the expert's optimal profit for all configurations of $(\alpha, \delta)$ with $(\alpha, \delta) \in$ $(0, \widehat{\alpha}) \otimes(0,1)$.

Proposition 4 There exists a unique cutoff $\alpha^{*} \in(0, \widehat{\alpha})$ such that
i) $\forall \alpha \in\left(0, \alpha^{*}\right]$, the maximum average profit is $\pi^{m}$ for $\delta \in\left[\underline{\delta}^{m}(\alpha), 1\right)$ and zero, otherwise;
ii) $\forall \alpha \in\left(\alpha^{*}, \bar{\alpha}\right]$, the maximum average profit is $\pi^{m}$ for $\delta \in\left[\underline{\delta}^{m}(\alpha), \underline{\delta}^{o}(\alpha)\right] \cup\left[\delta_{1}(\alpha), 1\right)$, $\pi^{o}$ for $\delta \in$ $\left(\underline{\delta}^{o}(\alpha), \delta_{1}(\alpha)\right)$ and zero, otherwise;
iii) $\forall \alpha \in(\bar{\alpha}, \widehat{\alpha})$, the maximum average profit is $\pi^{o}$ for $\delta \in\left[\underline{\delta}^{o}(\alpha), \delta_{1}(\alpha)\right)$, $\pi^{m}$ for $\delta \in\left[\delta_{1}(\alpha), 1\right)$ and zero,
otherwise.


Figure 1

We illustrated the maximum attainable average profit in Figure 1. The expert's maximum attainable average profit is $\pi^{m}$ in the red striped area and is $\pi^{o}$ in the blue shaded area with squares. It is zero for the remaining area.

When does the monitoring-by-rejection equilibrium dominate the one-price-fix-all equilibrium? Because the expert extracts the entire surplus from trade in both equilibria, the monitoring-by-rejection equilibrium is more profitable when it is more efficient than one-price-fix-all equilibrium. While the monitoring-by-rejection equilibrium involves undertreatment for the serious problem, the one-price-fix-all equilibrium involves overtreatment for the minor problem. The efficiency comparison depends on the likelihood of the serious problem as well as the expert's discount factor.

When consumers' problems are most likely to be minor (the parameter range $\left.\alpha \in\left(0, \alpha^{*}\right]\right)$, overtreatment for the minor problem is more costly than undertreatment for the serious problem. Consequently, the monitoring-by-rejection equilibrium dominates the one-price-fix-all equilibrium both in efficiency and profit.

Now, consider that the likelihood of the serious problem is relatively high (the parameter range $\alpha \in$
$(\bar{\alpha}, \widehat{\alpha}))$. When $\delta \geq \delta_{1}(\alpha), \pi^{m} \geq \pi^{o}$. This is because as $\delta$ converges to one, consumers' acceptance rate converges to one in the monitoring-by-rejection equilibrium, and hence the expert's profit converges to the first best. By contrast, the minor problem is always inefficiently repaired with probability one in the one-price-fix-all equilibrium, yielding $\pi^{o}$ dominated by $\pi^{m}$. As $\delta$ keeps decreasing, a larger rejection rate is necessary to discipline the expert to make honest recommendations, and the monitoring-by-rejection equilibrium becomes less efficient. As a result, when $\delta$ falls below $\delta_{1}(\alpha)$, the one-price-fix-all equilibrium dominates the monitoring-by-rejection equilibrium in efficiency and profit.

Finally, when $\alpha$ is in the intermediate range $\left(\alpha^{*}, \bar{\alpha}\right]$, something unusual happens. For high $\delta$, the monitoring-by-rejection equilibrium dominates. As $\delta$ falls below $\delta_{1}(\alpha)$, the one-price-fix-all equilibrium dominates the monitoring-by-rejection equilibrium in efficiency. However, as $\delta$ continues to fall below $\underline{\delta}^{o}(\alpha)$, the expert is no longer willing to fix the serious problem at a loss, causing the one-price-fix-all equilibrium to collapse. By contrast, the monitoring-by-rejection equilibrium is still sustainable, although giving the expert a lower profit. So the monitoring-by-rejection equilibrium dominates again.

Figure 1 provides some testable implications about the monopoly expert's conduct and market efficiency. For a given market, stronger expert reputation concerns are positively correlated with honesty, price and consumers' compliance rate. In addition, markets are more likely to involve overtreatment when consumers' problem are likely to result in a substantial loss (for example healthcare market) and undertreatment when their problems are likely to result in a minor loss (for example phone/computer repair services).

We close this section by reiterating the importance of studying credence expert services in a dynamic setting. Although credence goods and experience goods are not distinguishable in a static monopoly setting, the optimal equilibria in the repeated game are very different across these two markets. In the experience goods markets, a strong reputation concern can jointly support honest recommendation and efficient treatment. Market efficiency tends to jump and then stays flat after $\delta$ rises beyond a certain threshold. By contrast, in credence goods markets, market efficiency gradually increases in the expert's reputation concerns but never achieves the first best for any $\delta<1$.

## 5 Competitive market

In this section we consider a market with $n \geq 2$ experts and investigate how competition affects the monitoring technology for honesty and market efficiency. In the monopoly expert market, consumers monitor expert honesty by rejecting treatment recommendation which keeps the expert honest at the expense of burning some surplus from fixing the serious problem. When there are multiple experts in the marketplace, consumers can monitor expert honesty by soliciting second opinions and using conflicting recommendations to identify cheating experts. When an expert recommends the treatment for a consumer with the minor problem, he risks losing all future business if the consumer solicits a second opinion and the second opinion recommends no treatment. We explore this idea in this section.

We make minimal modifications to the monopoly model and retain its key elements, with the exception of adding the following new features. First, consumers pay a small search cost $k>0$ per visit except for their first visit. The assumption of a positive search cost captures the reality that search in expert markets often involves delay which is costly. No search cost for the first visit is inessential ${ }^{13}$ for our main results and, as pointed out by Stahl (1989), is "commonly assumed in the literature". This simplifying assumption allows us to make a fair comparison with the monopoly market which does not involve a cost of entry. Second, we allow entry and exit to maintain a stable pool of experts in the market in each period. When an expert exits the market, he is replaced by a new expert. Allowing entry and exit is realistic and makes our model tractable. Third, following the existing literature (Pesendorfer and Wolinsky (2003), Wolinsky (1993)), we assume that an expert cannot identify whether a consumer has visited another expert in the past.

Events in period $t$ unfold in the following sequence: Experts first simultaneously post treatment prices which become public information. A consumer arrives in the market and consults an expert. The expert learns the consumer's problem and makes a recommendation. The consumer either follows the expert's recommendation or goes on to search for another opinion. At the end of the period, experts' prices, the con-

[^8]sumer's utility and the recommendations made by all the experts she has visited become public information. If an expert exits the market, he is replaced by a new expert.

Next, we identify a class of equilibria in which consumers actively search for second opinions and use second opinions to monitor expert honesty for low search costs. We therefore call these equilibria in the following proposition "search equilibria".

Proposition 5 When $k$ is low, there exists a continuum of stationary search equilibria indexed by $p$. In each period, experts post the same price $p \in(c, c+k]$. They recommend the treatment for the serious problem with probability one and for the minor problem with probability $\beta_{m}^{*}(p ; k) \in(0,1)$. A consumer randomly visits an expert when arriving at the marketplace. If recommended the treatment on her first visit, the consumer accepts it with probability $\gamma^{*}(p ; k, \delta, n) \in(0,1)$ and searches for a second opinion with the complementary probability. If recommended the treatment again on her second visit, the consumer accepts the recommendation with probability one. Whenever the consumer is recommended no treatment, she exits the market. When the consumer receives conflicting recommendations, the expert who recommends the treatment loses all future business and exits the market. This equilibrium is sustainable for $\delta \geq \underline{\delta}(p ; k, n) \equiv \frac{n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)}{n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)+\alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)}$.

To understand a search equilibrium indexed by $p$, we first analyze a consumer's incentives to search for second opinions. When all of the experts lie with positive probabilities, the consumer may receive different recommendations from different experts and hence has incentives to search for second opinions. If the consumer accepts the treatment on her first visit, her problem will be fixed and she receives a payoff $-p$. Alternatively, the consumer can go on to search for a second opinion. If the second opinion recommends no treatment, the consumer infers that she must have the minor problem and hence will choose not to repair it. So, the consumer has a net benefit of $p-l_{m}$ from search when the second opinion contradicts the first opinion. For the consumer to randomize between accepting treatment and searching for a second opinion, it requires that the consumer's expected net benefit from search equals her search cost $k$, which gives

$$
\begin{equation*}
\operatorname{Pr}(\emptyset \mid p)\left(p-l_{m}\right)=k \tag{3}
\end{equation*}
$$

where $\operatorname{Pr}(\emptyset \mid p)$ is the probability that the second opinion recommends no treatment conditional on the first opinion recommending the treatment. When (3) holds, the consumer strictly prefers accepting the second
treatment recommendation to searching for third opinion. This is because the likelihood that the third opinion recommends no treatment conditional on two treatment recommendations is smaller than $\operatorname{Pr}(\emptyset \mid p)$, and hence the consumer's expected net benefit from search is strictly less than the search cost.

For the consumer to be willing to accept the treatment on her first visit, her expected loss must be at least as high as the price. So, the participation constraint is

$$
\begin{equation*}
p \leq \operatorname{Pr}\left(l_{s} \mid p\right) l_{s}+\operatorname{Pr}\left(l_{m} \mid p\right) l_{m} \tag{4}
\end{equation*}
$$

where $\operatorname{Pr}\left(l_{i} \mid p\right), i=m, s$, is the probability that the consumer's problem is $i$ conditional on a treatment recommendation. If the second opinion also recommends the treatment, the consumer updates her belief of having the serious problem upward and expects a greater loss from the problem. So, the participation constraint (4) implies that the consumer strictly prefers to accept the treatment on her second visit.

Now, we turn to analyze experts' recommendation strategies. An expert is indifferent between whether or not to recommend the treatment to the minor problem when the following condition holds:

where $\operatorname{Pr}\left(\mathrm{e} \mid l_{m}\right) \equiv \frac{\beta_{m}(1-\gamma)}{1+\beta_{m}(1-\gamma)}$ is the probability that the consumer is on her second visit conditional on her problem being minor and $V$ is the expert's continuation profit when he is active in the market. When an expert recommends the treatment for the minor problem, the treatment is accepted if the consumer is on her second visit or if she is on her first visit and chooses to accept the treatment, which happens with probability $\gamma$. The expert's profit margin from fixing the minor problem is $p-c+\varepsilon$. So, his expected current gain from recommending the treatment for the minor problem is on the right-hand side of (5). On the other hand, the expert risks losing all future business if he is caught lying. This happens when the consumer is on her first visit and decides to solicit a second opinion, and the second opinion happens to be honest. The expert's discounted expected future loss from lying is therefore given by the left-hand side of (5). When (5) holds, it is the expert's best response to randomize between recommending and not recommending treatment for the minor problem.

Finally, we construct consumers' off-equilibrium beliefs to prevent a price deviation. We assume consumers believe that an expert will recommend the treatment to both types of problems if he deviates to a price $p^{\prime}>c$ and will recommend the treatment only to the minor problem for $p^{\prime} \leq c$. Given this offequilibrium belief, a consumer will not be attracted to the deviating expert on her first visit. The expert can potentially offers a low price to attract consumers who search for second opinions, but $p \leq c+k$ prevents such a price deviation. To see this, given the off-equilibrium belief, consumers will visit the deviant for a second opinion only if $c<p^{\prime}<p$. Since the consumer expects the deviant to always recommend the treatment at $p^{\prime}$, she prefers to seek treatment from the deviant rather than accept the first treatment recommendation if and only if $p^{\prime}<p-k$. However, since $p \leq c+k, p^{\prime}<c$, which contradicts $c<p^{\prime}$. So, for $p \in(c, c+k]$, there does not exist a price deviation which can profitably attract consumers given the constructed off-equilibrium belief.

For a given price $p \in(c, c+k]$, there exists a search equilibrium which satisfies conditions (3), (4) and (5) when $k$ is sufficiently low and $\delta \geq \underline{\delta}(p ; k, n)$. To see that the discount factor must be high enough to sustain the search equilibrium, suppose $\delta$ is zero. Then, the expert does not bear any future loss from lying. Nevertheless, he has a positive current gain from lying because there is a positive probability that the consumer is on her second visit and will accept the treatment recommendation with probability one. So, the expert will lie with probability one when he does not have enough concerns for future business.

Contrary to the conventional wisdom that second opinions improve honesty ${ }^{14}$, we find the availability of second opinions may induce more dishonest recommendations. Recall that the monopoly expert makes honest recommendations when he is sufficiently patient or consumers are more likely to suffer a minor loss from their problems. By contrast, the search equilibrium involves small but pervasive cheating among experts. This is because some lying is necessary to induce consumers to search. In other words, full honesty is incompatible with search of second opinions: If experts all make honest recommendations, consumers will not pay the search cost to sample more opinions, rendering monitoring by second opinions ineffective.

Lemma 3 Consider an equilibrium indexed by $p \in(c, c+k)$. Holding $k$ constant, when $n$ increases, $\beta_{m}^{*}(p ; k)$

[^9]stays constant, $\underline{\delta}(p ; k, n)$ increases, $\gamma^{*}(p ; k, \delta, n)$ decreases, and experts' profits also decrease. Holding $n$ constant, when $k$ decreases, $\beta_{m}^{*}(p ; k)$ and $\underline{\delta}(p ; k, n)$ decrease, $\gamma^{*}(p ; k, \delta, n)$ increases, and experts' profits stay constant. Finally, both $\gamma^{*}(p ; k, \delta, n)$ and experts' profits increase in $\delta$.

Policy makers can make the market more competitive by increasing the number of experts or by decreasing the search cost $k$. Lemma 3 shows these two instruments to influence the competitiveness of the expert market have very different implications on honesty, search frequency and efficiency. As the number of experts in the market increases, it is harder to sustain the search equilibrium, consumers will solicit second opinions more frequently, and the honesty of experts' recommendation remain unchanged. This suggest that the search equilibrium becomes less efficient as $n$ increases because the equilibrium involves more costly search. To see the intuition, note that an expert's current gain from making an unnecessary treatment recommendation does not depend on the number of experts. However, the expert has less to lose when caught cheating when he has more competitors because the expert has a smaller market share. Consequently, it is harder to sustain the search equilibrium as $n$ increases. Moreover, consumers search more often in equilibrium when $n$ increases. Recall that the expert has less to lose from cheating when $n$ increases, so in order to make the expert indifferent between whether or not to cheat, his gain from cheating must be reduced proportionally, which requires consumers to reject the first treatment recommendation and solicit second opinions more often.

By contrast, when the search cost $k$ is reduced, experts become more honest, consumers solicit second opinions less frequently, and it is easier to sustain the equilibrium. Moreover, the equilibrium is more efficient because the overtreatment is reduced and consumers incur less costly search. When the search cost is reduced, a consumer's gain from seeking second opinions must also be reduced proportionally for the indifference condition (3) to hold. So, the probability that the second opinion contradicts the first opinion must be reduced, which requires the cheating probability $\beta_{m}^{*}(p ; k)$ to decrease. When experts become more honest, consumers will solicit second opinions less frequently in equilibrium. Our analysis shows that in markets for expert services, reducing search cost can improve recommendation honesty and treatment efficiency whereas increase in the number of experts is less effective and could even impede trade because it
is harder to sustain the search equilibria. Therefore, policies aiming to reduce consumers' costs of seeking second opinions are likely to improve welfare.

In a one-shot game, Wolinsky (1993) and Dulleck and Kerschbamer (2006) identify a class of equilibria in which experts randomize between whether or not to lie and consumers randomize between whether or not to seek second opinions. Despite the similar equilibrium behavior in their models and in ours, the driving forces for the equilibria are very different, yielding important differences in comparative statics. In Wolinsky (1993) and Dulleck and Kerschbamer (2006), short-lived experts have two treatments and it is efficient to use the expensive treatment for the serious problem and the inexpensive treatment for the minor problem. When deciding whether to recommend the expensive treatment for the minor problem, an expert trades off a high profit margin against a low acceptance rate because the expensive treatment is sometimes rejected and the inexpensive treatment is always accepted. By contrast, in our setting, experts have only one treatment and it is a dominant strategy for them to recommend the treatment for the minor problem in a static setting. Hence, the search equilibrium is not sustainable for short-lived experts. A long-lived expert trades off a current gain from recommending unnecessary services against a future loss of business. Because an expert's degree of patience and the number of experts in each period affect the value of the expert's future business, they also affect consumers' equilibrium search frequency $\gamma^{*}(p ; k, \delta, n)$ as in Corollary 3. By contrast, in Wolinsky (1993) and Dulleck and Kerschbamer (2006), consumers' search frequency is independent of experts' patience and the number of experts in the market.

Next, we investigate how search equilibria change in the search cost $k$ at the limit. Let $\bar{p} \equiv c+k$ denote the upper bound of the price of a search equilibrium.

Proposition 6 For a given $(\delta, n)$, when $k \rightarrow 0, \bar{p} \rightarrow c, \beta_{m}^{*}(p ; k) \rightarrow 0, \gamma^{*}(p ; k, \delta, n) \rightarrow 0, \underline{\delta}(p ; k, n) \rightarrow$ $\underline{\delta}^{c}(\alpha, n) \equiv \frac{n \alpha \varepsilon}{n \alpha \varepsilon+\alpha(1-\alpha)\left(c-l_{m}\right)}$, and the surplus of the search equilibrium converges to the first best $\alpha\left(l_{s}-c\right)$.

When the search cost is reduced, a consumer's gain from seeking second opinions must also be reduced proportionally for the indifference condition (3) to hold. So, when the search cost vanishes, the probability that the second opinion contradicts the first opinion must also vanish, which requires the cheating probability
$\beta_{m}^{*}(p ; k)$ to converge to zero. As the search cost diminishes, the cutoff discount factor $\underline{\delta}(p ; k, n)$ is affected by two opposing forces. On the one hand, an expert's continuation profit $V$ converges to zero as the price converges to $c$, so he has less to lose when caught cheating. This increases $\underline{\delta}(p ; k, n)$. On the other hand, the comparative statics $\gamma^{*}(p ; k, \delta, n) \rightarrow 0$ and $\beta_{m}^{*}(p ; k) \rightarrow 0$ suggests that cheating will be caught almost surely because the probability that a consumer will solicit an honest second opinion converges to one. This reduces $\underline{\delta}(p ; k, n)$. In the limit, the minimum discount factor necessary to support the search equilibrium converges to $\underline{\delta}^{c}(\alpha, n)$.

Proposition 6 implies that competition improves efficiency when the search cost is sufficiently low. While the optimal monopoly equilibrium converges to the first best only when $\delta$ converges to 1 , the search equilibrium converges to the first best as the search cost converges to zero even when the discount factor is bounded away by 1 . In our model, the efficiency gain from competition is due to the use of a more efficient monitoring technology for honesty. In the monopoly market, the serious problem must be left unrepaired sometimes for consumers to monitor expert honesty and the unrepaired serious problem can result in a big loss of efficiency. By contrast, this inefficiency from unrepaired serious problem is avoided in the search equilibrium. A skeptical consumer searches for second opinions instead of bearing the loss of her problem, so the serious problem is always repaired.

Finally, there are some interesting differences between the role of competition in credence goods and experience goods markets in the dynamic setting. First, in standard models of experience goods, competition typically impacts sellers' incentive through price effect, whereas in our analysis competition affects honesty and efficiency in credence goods markets through providing an alternative monitoring technology for honesty. In fact, search for second opinions is not considered in standard models of experience goods, but it is the key to more efficient monitoring in our analysis. Second, for experience goods, seller's honesty fully captures the quality and welfare. By contrast, in credence goods, honesty and efficiency do not always go hand in hand. We show that less honesty can be associated with more efficiency.

## 6 Discussion

In the main model, we focus on the parameter range $E(l)<c$ which implies $\alpha<\widehat{\alpha} \equiv \frac{c-l_{m}}{l_{s}-l_{m}}$. In this section, we discuss how the results change when $E(l) \geq c$, i.e. $\alpha \geq \widehat{\alpha}$.

Lemma 4 When $\alpha \geq \widehat{\alpha}$, the static game has a unique subgame perfect Nash equilibrium not involving weakly dominated strategies. In the equilibrium, the expert posts the price $p=E(l)$ and always recommends the treatment. Consumers accept the treatment recommendation with probability one.

In the static game, each possible price is followed by a proper subgame. In subgames following $p \geq c$, it is the expert's weakly dominant strategy to always recommend the treatment. Given that the expert will always recommend the treatment, it is the consumer's best response to accept the recommendation as long as $p \leq E(l)$. In subgames following $p<c$, the consumer will reject the expert's recommendation for the same argument outlined for Lemma 1. Hence, Lemma 4 is the unique SPNE not involving weakly dominated strategies. The static equilibrium involves overtreatment for the minor problem.

Lemmas 1 and 4 suggest that trade collapses in the static game if and only if the likelihood of the minor problem is sufficiently high. This is consistent with the equilibrium in the textbook lemon market model, which shows that market collapses when the proportion of "lemon" is high enough. In our model, the minor problem is the "lemon" whereas the serious problem is the "peach".

Next, we argue that the main findings of our analysis continue to hold qualitatively when $E(l) \geq c$. Similar as in Section 3, the static equilibrium in Lemma 4 holds both for credence goods and experience goods because the consumer's post consumption experience does not play any role in the one-shot game. When $\alpha \geq \widehat{\alpha}$, the static equilibrium resembles the one-price-fix-all equilibrium in Proposition 2. So, the one-price-fix-all equilibrium holds in the repeated game for any $\delta \in[0,1)$. How about the monitoring-byrejection equilibrium? It continues to exist when the expert is sufficiently patient, but the minimum discount factor required to sustain the equilibrium is higher than the case of $\alpha<\widehat{\alpha}$. This is because the static Nash equilibrium is more profitable than the static equilibrium under the assumption $\alpha<\widehat{\alpha}$ and therefore the expert has less to lose when caught cheating, which makes it harder to sustain the equilibrium. Nevertheless,
when $\delta$ is sufficiently close to one, the monitoring-by-rejection equilibrium approaches to the first best. Finally, for the competitive setting, the search equilibrium continues to exist when $\delta$ is sufficiently high and $k$ is sufficiently low because the equilibrium strategy does not depend on the assumption of $E(l)<c$.

## 7 Concluding remarks

This paper studies trust building in a dynamic setting of credence professional services. In these markets, due to their lack of expertise, consumers cannot assess the value of the services provided by experts even after receiving the services, and hence they cannot monitor expert honesty through their consumption experience. This stands in sharp contrast with experience goods markets in which consumers learn and monitor the quality of experience goods by consumption.

In the monopoly credence-goods market, expert honesty is monitored by consumer rejection of treatment recommendations. The efficiency of the optimal equilibrium gradually increases in the expert's discount factor but never achieves the first best. The optimal equilibrium either involves undertreatment for the serious problem or overtreatment for the minor problem. This contrasts sharply with experience goods markets in which the first best is achieved when the seller's discount factor is greater than a threshold.

When there is competition, consumers can use second opinions to monitor expert honesty. As the search cost converges to zero, the competitive equilibrium converges to the first best. Nevertheless, the competitive equilibrium still involves some inefficiency as long as the search cost is positive. Although competition may improve market efficiency, it comes at a cost of less honesty when the discount factor is high.

Our study suggests that reputation concerns are less effective in improving efficiency in credence goods markets than in experience goods markets. Moreover, in credence goods markets wherein experts do not have very strong reputation concerns, competition can improve market efficiency via consumer search for second opinions when search costs are low. So, reduction of search costs and encouragement of consumers' solicitation of second opinions should be an integral part of any regulatory policies aiming at enhancing welfare.

## Appendix A

Proof for Lemma 1: We confine the analysis to the case $p \geq c-\varepsilon$ because it is weakly dominated for the expert to recommend a treatment at $p<c-\varepsilon$. Suppose that trade happens with a positive probability, then $p$ must be at least $c$. To see this, if $p<c$, the expert will recommend the treatment only when the consumer has the minor problem. Since $l_{m}<c-\varepsilon \leq p$, the consumer's best response is to reject $p$ with probability one. Now, consider $c \leq p$. If $p$ is accepted with a positive probability, the expert will recommend $p$ for the minor problem with probability one. This implies $E(l \mid p) \leq E(l)$, where $E(l \mid p)$ is the consumer's expected loss from the problem, conditional on being recommended the treatment $p$. But then it is the consumer's best response to reject $p$ with probability one because $E(l \mid p) \leq E(l)<c \leq p$. A contradiction. Q.E.D.

Proof for Proposition 1: Given the expert's strategy, a consumer's expected loss from her problem is $l_{s}$ upon a treatment recommendation. So, the consumer is indifferent between accepting or rejecting the treatment offer and therefore it is her best response to accept the treatment offer with probability $\gamma^{*}$.

Given price $p=l_{s}$, the expert does not have a profitable deviation in his recommendation strategy. The expert will make a positive profit from recommending the treatment for the serious problem but will make zero profit if she recommends no treatment to the problem. Hence, the expert does not have an incentive to refuse to treat the serious problem. Consider that the consumer has the minor problem. The condition (2) holds at $\gamma^{*}$, which makes the expert just indifferent between whether or not to recommend the treatment. So, it is the expert's best response to recommend no treatment for the minor problem. Last, for $\gamma^{*} \geq 0$, it is necessary to have $\delta \geq \underline{\delta}^{m}$.

Finally, the expert does not have a profitable price deviation. A price deviation outside the range $\left[c-\varepsilon, l_{s}\right)$ is not profitable because the expert will make a loss from repairing a problem at $p^{\prime}<c-\varepsilon$ and the consumer will reject a price $p^{\prime}>l_{s}$ irrespective of her belief about her problem. Consumers will reject a price deviation in the range of $\left[c-\varepsilon, l_{s}\right)$, given the off-equilibrium belief specified for Proposition 1 in the main text. Hence, any price deviation leads to zero profit because the expert's recommendation will be rejected in the current period and the deviation will trigger the reversion to the punishment path from the next period onward. Q.E.D.

Proof for Proposition 2: consumers' maximum willingness to pay in a one-price-fix-all equilibrium is $E(l)$. Hence, it is consumers' best response to accept the expert's treatment offer with probability one. The game reverts to the static Nash equilibrium perpetually if the expert refuses to treat a consumer or deviates to a price different from $E(l)$.

First, consider the expert's incentives to deviate in the case of the serious problem. The expert's profit from fixing the serious problem is

$$
E(l)-c+\frac{\delta \pi}{1-\delta}
$$

where $\pi=E(l)-c+(1-\alpha) \varepsilon$. The expert receives zero profit if he refuses to fix the consumer's problem because all players revert to the static Nash equilibrium from the next period onward. So, the no deviation condition requires

$$
\begin{align*}
E(l)-c+\frac{\delta \pi}{1-\delta} & \geq 0  \tag{6}\\
\delta & \geq \underline{\delta}^{o} \equiv \frac{c-E(l)}{(1-\alpha) \varepsilon}
\end{align*}
$$

The cutoff $\underline{\delta}^{o}$ is positive given the assumption $E(l)<c$. It can be verified that $\underline{\delta}^{o}<1$ if and only if $\alpha \geq \widetilde{\alpha}$. Now consider the expert's no deviation incentives when the consumer's problem is minor:

$$
\begin{equation*}
E(l)-c+\varepsilon+\frac{\delta \pi}{1-\delta} \geq 0 \tag{7}
\end{equation*}
$$

where the left hand side of (7) is the expert's payoff from repairing the minor problem. Condition (6) implies (7) because it is less costly to repair the minor problem than the serious problem. As a result, the expert does not have a profitable deviation in his recommendation strategy when $\delta \geq \underline{\delta}^{o}$.

Finally, assume that consumers hold the same off-equilibrium belief following a price deviation as in the proof for Proposition 1. Given this off-equilibrium belief, a price different from $E(l)$ will be rejected in the current period and result in zero future profit. Thus, the expert does not have a profitable deviation in price. Q.E.D.

Proof for Lemma 2: Substitute $\underline{\delta}^{m}(\alpha)$ and $\underline{\delta}^{o}(\alpha)$ defined in Propositions 1 and 2 and take the difference:

$$
\underline{\delta}^{o}(\alpha)-\underline{\delta}^{m}(\alpha)=\frac{-\alpha^{2}\left(l_{s}-l_{m}\right)\left(l_{s}-c\right)-\alpha\left[\left(c-\varepsilon-l_{m}\right) \varepsilon+\left(l_{s}-c\right)^{2}\right]+\left(c-\varepsilon-l_{m}\right)\left(l_{s}-c+\varepsilon\right)}{(1-\alpha)\left[\varepsilon+(1+\alpha)\left(l_{s}-c\right)\right] \varepsilon}
$$

Since the denominator is positive, the sign of $\underline{\delta}^{o}(\alpha)-\underline{\delta}^{m}(\alpha)$ is determined by the numerator which we denote by $f(\alpha)$. The derivative of $f(\alpha)$ is

$$
f^{\prime}(\alpha)=-2 \alpha\left(l_{s}-l_{m}\right)\left(l_{s}-c\right)-\left[\left(c-\varepsilon-l_{m}\right) \varepsilon+\left(l_{s}-c\right)^{2}\right]<0
$$

So, $f(\alpha)$ is strictly decreasing in the range of $(\widetilde{\alpha}, \widehat{\alpha})$.

Next, we show $f(\widetilde{\alpha})>0$. To see this, note that $\underline{\delta}^{o}(\widetilde{\alpha})=1$ and $\underline{\delta}^{m}(0)=1$. By Corollary $1, \underline{\delta}^{m}(\alpha)$ is strictly decreasing in $\alpha$, and hence $\underline{\delta}^{m}(\widetilde{\alpha})<\underline{\delta}^{o}(\widetilde{\alpha})=1$.

Now, we evaluate $f(\alpha)$ at $\alpha=\widehat{\alpha} \equiv \frac{c-l_{m}}{l_{s}-l_{m}}$ and obtain $f(\widehat{\alpha})=\frac{-\left(l_{s}-c\right)\left(l_{s}-c+\varepsilon\right) \varepsilon}{l_{s}-l_{m}}<0$. Given that $f(\alpha)$ is continuous and decreasing and that $f(\widetilde{\alpha})>0$ and $f(\widehat{\alpha})<0$, there exists a unique $\bar{\alpha} \in(\widetilde{\alpha}, \widehat{\alpha})$, such that $\underline{\delta}^{o} \geq \underline{\delta}^{m}$ for $\alpha \in(\widetilde{\alpha}, \bar{\alpha}]$ and $\underline{\delta}^{o}<\underline{\delta}^{m}$ for $\alpha \in(\bar{\alpha}, \widehat{\alpha})$. Q.E.D.

Proof for Proposition 4: The proof is divided into 4 steps. Step 1 defines $\alpha^{*}$ and characterizes properties of $\alpha^{*}$ and $\delta_{1}(\alpha)$, which will be used for the subsequent steps. Steps 2,3 and 4 prove cases i), ii) and iii) in the Proposition, respectively. Figure 1 will facilitate the understanding of the proof.

Step 1. Define $\alpha^{*}$ as the value which solves $\delta_{1}(\alpha)=\underline{\delta}^{o}(\alpha)$. We show that there exists a unique $\alpha^{*} \in(\widetilde{\alpha}, \bar{\alpha})$. To begin, we first prove that there exists a unique $\alpha^{*} \in(\widetilde{\alpha}, \widehat{\alpha})$. Evaluate $\delta_{1}(\alpha)$ and $\underline{\delta}^{o}(\alpha)$ at $\alpha=\widetilde{\alpha}$ and $\alpha=\widehat{\alpha}$, respectively. We have

$$
\begin{align*}
\delta_{1}(\widetilde{\alpha}) & =\frac{l_{s}-c+\varepsilon}{l_{s}-c+\varepsilon+(1-\widetilde{\alpha})\left(c-\varepsilon-l_{m}\right)}<1=\underline{\delta}^{o}(\widetilde{\alpha})  \tag{8}\\
\delta_{1}(\widehat{\alpha}) & =\frac{l_{s}-c+\varepsilon}{l_{s}-c+\varepsilon+(1-\widehat{\alpha})\left(c-\varepsilon-l_{m}\right)}>0=\underline{\delta}^{o}(\widehat{\alpha}) \tag{9}
\end{align*}
$$

Note that $\delta_{1}(\alpha)$ and $\underline{\delta}^{o}(\alpha)$ are continuous in the range of $(\widetilde{\alpha}, \widehat{\alpha})$ and $\delta_{1}(\alpha)$ is strictly increasing while $\underline{\delta}^{o}(\alpha)$ is strictly decreasing (Corollary 2). As a result, conditions (8) and (9) implies that there is a unique $\alpha^{*} \in(\widetilde{\alpha}, \widehat{\alpha})$ such that $\delta_{1}(\alpha)>\underline{\delta}^{o}(\alpha)$ for $\alpha>\alpha^{*}$ and $\delta_{1}(\alpha) \leq \underline{\delta}^{o}(\alpha)$ for $\alpha \leq \alpha^{*}$.

Next, we show $\alpha^{*}<\bar{\alpha}$ by contradiction. Suppose $\alpha^{*} \geq \bar{\alpha}$. Then, it follows that

$$
\begin{equation*}
\underline{\delta}^{m}(\bar{\alpha})=\underline{\delta}^{o}(\bar{\alpha}) \geq \underline{\delta}^{o}\left(\alpha^{*}\right)=\delta_{1}\left(\alpha^{*}\right) \geq \delta_{1}(\bar{\alpha}) \tag{10}
\end{equation*}
$$

The equalities follows from the definitions of $\bar{\alpha}$ and $\alpha^{*}$, respectively. The first inequality holds because $\underline{\delta}^{o}(\alpha)$ strictly decreases in $\alpha$ and the second inequality holds because $\delta_{1}(\alpha)$ strictly increases in $\alpha$. Furthermore, it
can be verified that $\underline{\delta}^{m}(\widetilde{\alpha})=\delta_{1}(\widetilde{\alpha})$. Since $\underline{\delta}^{m}(\alpha)$ strictly decreases in $\alpha$ while $\delta_{1}(\alpha)$ strictly increases in $\alpha$, $\underline{\delta}^{m}(\alpha)<\delta_{1}(\alpha)$ for all $\alpha>\widetilde{\alpha}$, which contradicts (10) given $\bar{\alpha}>\widetilde{\alpha}$.

Step 2. Consider $\alpha \in\left(0, \alpha^{*}\right]$. By Step $1, \alpha^{*}<\bar{\alpha}$, so $\underline{\delta}^{m}(\alpha)<\underline{\delta}^{o}(\alpha)$ for $\alpha \in\left(0, \alpha^{*}\right]$. The one-price-fix-for-all equilibrium is not sustainable if $\underline{\delta}^{m}(\alpha) \leq \delta<\underline{\delta}^{o}(\alpha)$ by Proposition 2. So, the monitoring-by-rejection equilibrium yields the expert the highest profit. Next, consider $\underline{\delta}^{o}(\alpha) \leq \delta<1$. By Step $1, \delta_{1}(\alpha) \leq \underline{\delta}^{o}(\alpha)$ for $\alpha \leq \alpha^{*}$. Consequently, $\pi^{o} \leq \pi^{m}$ for $\delta_{1}(\alpha) \leq \underline{\delta}^{o}(\alpha) \leq \delta$ by the definition of $\delta_{1}(\alpha)$.

Step 3. Consider $\alpha \in\left(\alpha^{*}, \bar{\alpha}\right]$. By Lemma $2, \underline{\delta}^{m}(\alpha)<\underline{\delta}^{o}(\alpha)$ for $\alpha<\bar{\alpha}$ and therefore the maximum average profit is $\pi^{m}$ for $\underline{\delta}^{m}(\alpha) \leq \delta \leq \underline{\delta}^{o}(\alpha)$. Now, consider $\underline{\delta}^{o}(\alpha)<\delta$. By Step $1, \delta_{1}(\alpha)>\underline{\delta}^{o}(\alpha)$ for $\alpha \in\left(\alpha^{*}, \bar{\alpha}\right]$. It follows that $\pi^{m} \leq \pi^{o}$ for $\underline{\delta}^{o}(\alpha) \leq \delta<\delta_{1}(\alpha)$ and $\pi^{m}>\pi^{o}$ for $\delta_{1}(\alpha) \leq \delta<1$.

Step 4. Consider $\alpha \in(\bar{\alpha}, \widehat{\alpha}]$. By Lemma $2, \underline{\delta}^{o}(\alpha)<\underline{\delta}^{m}(\alpha)$ for $\bar{\alpha}<\alpha$. So, the maximum average profit is $\pi^{o}$ for $\underline{\delta}^{o}(\alpha) \leq \delta<\underline{\delta}^{m}(\alpha)$. Both $\pi^{m}$ and $\pi^{o}$ can be supported for $\underline{\delta}^{m}(\alpha) \leq \delta<1$. Since $\underline{\delta}^{m}(\alpha)<\delta_{1}(\alpha)$ for $\alpha \in(\bar{\alpha}, \widehat{\alpha})$, by Step $1, \pi^{m} \geq \pi^{o}$ for $\delta_{1}(\alpha) \leq \delta<1$ and $\pi^{m}<\pi^{o}$ for $\underline{\delta}^{m}(\alpha) \leq \delta<\delta_{1}(\alpha)$. Q.E.D.

Proof for Proposition 5: Define

$$
\begin{align*}
\beta_{m}^{*}(p ; k) & \equiv \frac{(1-\alpha)\left(p-l_{m}-k\right)-\sqrt{(1-\alpha)^{2}\left(p-l_{m}-k\right)^{2}-4 k \alpha(1-\alpha)\left(p-l_{m}\right)}}{2(1-\alpha)\left(p-l_{m}\right)}  \tag{11}\\
\underline{\delta}(p ; k, n) & \equiv \frac{n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)}{n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)+\alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)},  \tag{12}\\
\gamma^{*}(p ; k, n) & \equiv 1-\frac{n(1-\delta)(p-c+\varepsilon)}{n(1-\delta)(p-c+\varepsilon)\left(1-\beta_{m}^{*}(p ; k)\right)+\delta \alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)} . \tag{13}
\end{align*}
$$

First, note that the expression under the square root in (11) is positive if $k \leq \frac{\left(c-l_{m}\right)(1-\sqrt{\alpha})}{1+\sqrt{\alpha}}$. So, $\beta_{m}^{*}(p ; k) \in$ $[0,1)$ for $k \leq \frac{\left(c-l_{m}\right)(1-\sqrt{\alpha})}{1+\sqrt{\alpha}}$.

Next, we show that consumers' strategy is a best response given experts' strategies. When a consumer receives a treatment recommendation on her first visit, the probability that the second opinion recommends no treatment is $\operatorname{Pr}(\emptyset \mid p)=\frac{(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)}{\alpha+(1-\alpha) \beta_{m}}$. Substitute $\operatorname{Pr}(\emptyset \mid p)$ into the search condition (3), it becomes

$$
\begin{equation*}
\frac{(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)\left(p-l_{m}\right)}{\alpha+(1-\alpha) \beta_{m}}=k \tag{14}
\end{equation*}
$$

It can be verified that (14) is satisfied at $\beta_{m}^{*}(p ; k)$ defined in (11). Note that the item in the square root in $\beta_{m}^{*}(p ; k)$ is positive and $\beta_{m}^{*}(p ; k) \in(0,1)$ when $k$ is sufficiently low.

Next, consider the consumer's participation constraint (4). Substitute $\operatorname{Pr}\left(l_{s} \mid p\right)=\frac{\alpha}{\alpha+(1-\alpha) \beta_{m}}$ and $\operatorname{Pr}\left(l_{m} \mid p\right)=$ $\frac{(1-\alpha) \beta_{m}}{\alpha+(1-\alpha) \beta_{m}}$ into (4), we obtain

$$
\begin{equation*}
p \leq \frac{\alpha l_{s}+(1-\alpha) \beta_{m} l_{m}}{\alpha+(1-\alpha) \beta_{m}} \tag{15}
\end{equation*}
$$

When $k$ converges to $0, \beta_{m}^{*}(p ; k)$ converges to 0 . So, the right hand side of (15) converges to $l_{s}$. Since $p \leq c+k$ and $c<l_{s}$, the participation constraint (15) is satisfied when $k$ is sufficiently small. We conclude that given $p$ and $\beta_{m}^{*}(p ; k)$, it is the consumer's best response to randomize between accepting the treatment on her first visit and searching for a second opinion.

Next, we show that it is the consumer's best response to accept the second treatment recommendation with probability one. The probability that the third opinion recommends no treatment conditional on the first two opinions recommending the treatment is denoted by $\operatorname{Pr}(\emptyset \mid p p)=\frac{(1-\alpha)\left(\beta_{m}\right)^{2}\left(1-\beta_{m}\right)}{\alpha+(1-\alpha)\left(\beta_{m}\right)^{2}}$. Because $\operatorname{Pr}(\emptyset \mid p p)<\operatorname{Pr}(\emptyset \mid p)$, it follows that $\operatorname{Pr}(\emptyset \mid p p)\left(p-l_{m}\right)<k$ and hence the consumer strictly prefers accepting the second treatment recommendation to searching for the third opinion. Finally, the participation constraint (15) is satisfied when the consumer receives the second treatment recommendation. Hence, it is the consumer 's best response to accept the second treatment recommendation with probability one.

Now, we show that experts' strategies are their best response. We show that experts have no profitable deviation in their recommendation strategies. When the expert diagnoses that a consumer has the minor problem, the probability that she is on her second visit is

$$
\begin{equation*}
\operatorname{Pr}\left(\mathrm{e} \mid l_{m}\right)=\frac{(1-\alpha) \frac{1}{n} \beta_{m}(1-\gamma)}{(1-\alpha)\left[\frac{1}{n}+\frac{1}{n} \beta_{m}(1-\gamma)\right]}=\frac{\beta_{m}(1-\gamma)}{1+\beta_{m}(1-\gamma)} \tag{16}
\end{equation*}
$$

Denote by $\Pi \equiv \alpha(p-c)+(1-\alpha)(p-c+\varepsilon)\left(\gamma \beta_{m}+(1-\gamma)\left(\beta_{m}\right)^{2}\right)$ the industry profit in a period. The expected industry profit from treating the serious problem is $\alpha(p-c)$ because the serious problem is repaired with probability one. The minor problem is fixed with probability $\gamma \beta_{m}+(1-\gamma)\left(\beta_{m}\right)^{2}$, where $\gamma \beta_{m}$ is the probability that the minor problem is fixed on the consumer's first visit and $(1-\gamma)\left(\beta_{m}\right)^{2}$ is the probability that it is fixed on her second visit. Let $V$ denote the present value of the expert's profit when he is active in the market. It follows that

$$
\begin{equation*}
V=\frac{\Pi}{n}+\delta\left[1-\frac{1}{n}(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)(1-\gamma)\right] V \tag{17}
\end{equation*}
$$

If the expert is active in a period, he receives $\frac{1}{n}$ of the industry profit in that period. The expert will lose all future consumers if he is caught lying in the current period, which occurs with probability $\frac{1}{n}(1-\alpha) \beta_{m}(1-$ $\left.\beta_{m}\right)(1-\gamma)$. So, the expert will survive to the next period with the complementary probability and his expected continuation payoff is the second item in (17). Solving (17), we obtain

$$
\begin{equation*}
V=\frac{\Pi}{n-\delta\left[n-(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)(1-\gamma)\right]}=\frac{\alpha(p-c)+(1-\alpha)(p-c+\varepsilon)\left(\gamma \beta_{m}+(1-\gamma)\left(\beta_{m}\right)^{2}\right)}{n-\delta\left[n-(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)(1-\gamma)\right]} \tag{18}
\end{equation*}
$$

Substituting $V$ and (16) into (5), it follows that

$$
\begin{align*}
& \delta(1-\gamma)\left(1-\beta_{m}\right) \frac{\alpha(p-c)+(1-\alpha)(p-c+\varepsilon)\left(\gamma \beta_{m}+(1-\gamma)\left(\beta_{m}\right)^{2}\right)}{n-\delta\left[n-(1-\alpha) \beta_{m}\left(1-\beta_{m}\right)(1-\gamma)\right]}  \tag{19}\\
= & (p-c+\varepsilon)\left[\beta_{m}(1-\gamma)+\gamma\right] .
\end{align*}
$$

The condition (19) pins down a unique solution $\gamma=1-\frac{n(1-\delta)(p-c+\varepsilon)}{n(1-\delta)(p-c+\varepsilon)\left(1-\beta_{m}\right)+\delta \alpha\left(1-\beta_{m}\right)(p-c)}$. So, given that all other experts choose the strategy $\left(p, \beta_{m}^{*}(p ; k)\right)$ and that the consumer chooses $\gamma=\gamma^{*}(p ; k, n), \beta_{m}^{*}(p ; k)$ is the expert's best response. Note that for $\gamma^{*}(p ; k, n)>0$, it is necessary to have $\delta>\underline{\delta}(p ; k, n)$. Q.E.D.

Proof for Lemma 3: We first show $\frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}>0$. Using (14), we derive

$$
\begin{equation*}
\frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}=\frac{\alpha+(1-\alpha) \beta_{m}^{*}(p ; k)}{(1-\alpha)\left(p-l_{m}\right)\left(1-2 \beta_{m}^{*}(p ; k)\right)-(1-\alpha) k} . \tag{20}
\end{equation*}
$$

So, the sign of $\frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}$ is determined by the sign of the denominator. Substituting (11) into the denominator of (20), we obtain

$$
(1-\alpha)\left[\left(p-l_{m}\right)\left(1-2 \beta_{m}^{*}(p ; k)\right)-k\right]=\sqrt{(1-\alpha)^{2}\left(p-l_{m}-k\right)^{2}-4 k \alpha(1-\alpha)\left(p-l_{m}\right)}>0
$$

Hence, $\frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}>0$. Since $\beta_{m}^{*}(p ; k)$ is independent of $n$ and $\delta, \frac{\partial \beta_{m}^{*}(p ; k)}{\partial n}=\frac{\partial \beta_{m}^{*}(p ; k)}{\partial \delta}=0$.

Next, using (13), we derive

$$
\begin{align*}
\frac{\partial \gamma^{*}(p ; k, \delta, n)}{\partial k} & =\frac{n(1-\delta)(p-c+\varepsilon)}{(n(1-\delta)(p-c+\varepsilon)+\delta \alpha(p-c))\left(1-\beta_{m}^{*}(p ; k)\right)^{2}} \frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}>0  \tag{21}\\
\frac{\partial \gamma^{*}(p ; k, \delta, n)}{\partial n} & =-\frac{\alpha \delta(1-\delta)\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)(p-c+\varepsilon)}{\left[n(1-\delta)(p-c+\varepsilon)\left(1-\beta_{m}^{*}(p ; k)\right)+\delta \alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)\right]^{2}}<0  \tag{22}\\
\frac{\partial \gamma^{*}(p ; k, \delta, n)}{\partial \delta} & =\frac{\alpha n\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)(p-c+\varepsilon)}{\left[n(1-\delta)(p-c+\varepsilon)\left(1-\beta_{m}^{*}(p ; k)\right)+\delta \alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)\right]^{2}}>0 \tag{23}
\end{align*}
$$

Following (12), we obtain

$$
\begin{aligned}
& \frac{\partial \underline{\delta}(p ; k, n)}{\partial k}=\frac{n \alpha(p-c)(p-c+\varepsilon)}{\left[n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)+\alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)\right]^{2}} \frac{\partial \beta_{m}^{*}(p ; k)}{\partial k}>0 . \\
& \frac{\partial \underline{\delta}(p ; k, n)}{\partial n}=\frac{\alpha \beta_{m}^{*}(p ; k)\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)(p-c+\varepsilon)}{\left[n(p-c+\varepsilon) \beta_{m}^{*}(p ; k)+\alpha\left(1-\beta_{m}^{*}(p ; k)\right)(p-c)\right]^{2}}>0
\end{aligned}
$$

Last, we investigate the impact of $k, n$ and $\delta$ on experts' profits. Substituting (13) into (18) and simplifying the expression, we obtain $V=\frac{\alpha(p-c)}{n(1-\delta)}$. Hence, $\frac{\partial V}{\partial k}=0, \frac{\partial V}{\partial n}<0$ and $\frac{\partial V}{\partial \delta}>0$. Q.E.D.

Proof for Proposition 6: Consider the search equilibrium indexed by the price $p^{*}=c+k$. Substituting $p^{*}=c+k$ into (11) and (13), it directly follows that $\beta_{m}^{*}(c+k ; k) \rightarrow 0$ and $\gamma^{*}(c+k ; k, n) \rightarrow 0$ as $k \rightarrow 0$. At $p^{*}=c+k$, the cutoff discount factor is

$$
\underline{\delta}(c+k ; k, n) \equiv \frac{n(k+\varepsilon) \beta_{m}^{*}(c+k ; k)}{n(k+\varepsilon) \beta_{m}^{*}(c+k ; k)+\alpha\left(1-\beta_{m}^{*}(c+k ; k)\right) k} .
$$

Since both the numerator and the denominator of $\underline{\delta}(c+k ; k, n)$ converges to 0 as $k \rightarrow 0$, we apply L'Hospital's Rule

$$
\begin{align*}
\lim _{k \rightarrow 0} \underline{\delta}(c+k ; k, n) & =\lim _{k \rightarrow 0} \frac{n \beta_{m}^{*}(c+k ; k)+n(k+\varepsilon) \partial \beta_{m}^{*}(c+k ; k) / \partial k}{n \beta_{m}^{*}(c+k ; k)+n(k+\varepsilon) \partial \beta_{m}^{*}(c+k ; k) / \partial k+\alpha\left(1-\beta_{m}^{*}(c+k ; k)\right)-\alpha k \partial \beta_{m}^{*}(c+k ; k)(24)} \\
& =\frac{n \varepsilon \lim _{k \rightarrow 0} \partial \beta_{m}^{*}(c+k ; k) / \partial k}{n \varepsilon \lim _{k \rightarrow 0} \partial \beta_{m}^{*}(c+k ; k) / \partial k+\alpha} \tag{25}
\end{align*}
$$

where the first equality follows from L'Hospital's Rule and the second equality follows from $\beta_{m}^{*}(c+k ; k) \rightarrow 0$ as $k \rightarrow 0$.

Next, we derive $\lim _{k \rightarrow 0} \partial \beta_{m}^{*}(c+k ; k) / \partial k$. Let $N(k)$ denote the numerator of $\beta_{m}^{*}(c+k ; k)$ and $D(k)$ denote the denominator of $\beta_{m}^{*}(c+k ; k)$. So,

$$
\begin{equation*}
\partial \beta_{m}^{*}(c+k ; k) / \partial k=\frac{N^{\prime}(k) D(k)-N(k) D^{\prime}(k)}{(D(k))^{2}} \tag{26}
\end{equation*}
$$

The derivatives $N^{\prime}(k)$ and $D^{\prime}(k)$ are

$$
\begin{aligned}
N^{\prime}(k) & =\frac{2 \alpha(1-\alpha)\left(c+2 k-l_{m}\right)}{\sqrt{(1-\alpha)^{2}\left(c-l_{m}\right)^{2}-4 k \alpha(1-\alpha)\left(c+k-l_{m}\right)}} \\
D^{\prime}(k) & =2(1-\alpha)
\end{aligned}
$$

Substitute $N^{\prime}(k)$ and $D^{\prime}(k)$ into (26) and take the limit,

$$
\begin{equation*}
\lim _{k \rightarrow 0} \partial \beta_{m}^{*}(c+k ; k) / \partial k=\lim _{k \rightarrow 0} \frac{N^{\prime}(k) D(k)-N(k) D^{\prime}(k)}{(D(k))^{2}}=\frac{\alpha}{(1-\alpha)\left(c-l_{m}\right)}, \tag{27}
\end{equation*}
$$

which follows because $\lim _{k \rightarrow 0} D(k)=2(1-\alpha)\left(c-l_{m}\right), \lim _{k \rightarrow 0} D^{\prime}(k)=2(1-\alpha), \lim _{k \rightarrow 0} N(k)=0$, and $\lim _{k \rightarrow 0} N^{\prime}(k)=2 \alpha$. Substitute (27) into (25), it follows that $\lim _{k \rightarrow 0} \delta(c+k ; k, n)=\frac{n \alpha \varepsilon}{n \alpha \varepsilon+\alpha(1-\alpha)\left(c-l_{m}\right)}$. Q.E.D.

## Appendix B

We use the method proposed by APS (1990) and elaborated in Mailath and Samuleson (2006) to characterize the maximum self-generation set. Since we allow public randomization at the end of each period of the game, the set of the expert's payoff in any Perfect Public equilibria is an interval $[0, \bar{v}]$, where 0 is the expert's payoff in the static game and $\bar{v}$ is the highest possible obtainable profit in a Perfect Public equilibrium. We characterize the largest self-generating set in the following two lemmas. Lemma 5 considers the price range $p \geq c$ and Lemma 6 considers the price range $p<c$. We first denote the five public outcomes $y \in Y \equiv\{0,1,2,3,4\}$ described in the paragraph following Proposition 3 for future use:

$$
y=\left\{\begin{array}{ccc}
0 & \text { if } & R=\emptyset, l=l_{m} \\
1 & \text { if } & R=\emptyset, l=l_{s} \\
2 & \text { if } & R=p, a=1 \\
3 & \text { if } & R=p, a=0, l=l_{m} \\
4 & \text { if } & R=p, a_{t}=0, l=l_{s}
\end{array} .\right.
$$

Lemma 5 When $p \geq c, \bar{v}=\pi^{m}$ for $\delta \geq \underline{\delta}^{m}(\alpha)$ and $\bar{v}=0$ For $\delta<\underline{\delta}^{m}(\alpha)$.

Since clients are myopic, we focus on the expert's value set. Note that when $\beta_{s} \leq \beta_{m}$, upon a treatment recommendation at $p \geq c$, the client's expected loss is at most $E(l)<c \leq p$. So a short-lived consumer's best response is $\gamma=0$. Let $\Omega_{1} \equiv\left\{p,\left\{\beta \mid 0 \leq \beta_{s} \leq \beta_{m} \leq 1\right\}, \gamma=0\right\}$ and $\Omega_{2} \equiv\left\{p,\left\{\beta \mid 0 \leq \beta_{m}<\beta_{s} \leq 1\right\}, \gamma \in[0,1]\right\}$. The action profiles involving short-lived clients' best response are thus $\Omega_{1} \cup \Omega_{2}$. The expert's continuation payoff is a mapping $v: Y \rightarrow[0, \bar{v}]$. Let $V^{i}, i=1,2$, denote the set of payoffs decomposed by $\Omega_{i}$ on $[0, \bar{v}]$.

The proof is divided into three steps. Step 1 characterizes $V^{1}$. Step 2 characterizes $V^{2}$. Step 3 characterizes the maximum self-generating set.

Step 1 Given $\gamma=0$, the expert's expected payoff is

$$
v=\alpha \delta\left[\beta_{s} v(4)+\left(1-\beta_{s}\right) v(1)\right]+(1-\alpha) \delta\left[\beta_{m} v(3)+\left(1-\beta_{m}\right) v(0)\right] .
$$

Since this strategy profile constitute an equilibrium in the stage game, it is trivially enforceable using a
constant continuation $v(4)=v(1)=(3)=v(0)=\widehat{v} \in[0, \bar{v}]$. Thus, $V^{1}=[0, \delta \bar{v}]$, where the upper bound is achieved by $\widehat{v}=\bar{v}$ and the lower bound is achieved by $\widehat{v}=0$.

Step 2 Given the action profile, the expert's expected payoff is

$$
\begin{align*}
v \equiv & \alpha\left\{\beta_{s}[\gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4)]+\left(1-\beta_{s}\right) \delta v(1)\right\}+  \tag{28}\\
& (1-\alpha)\left\{\beta_{m}[\gamma((1-\delta)(p-c+\varepsilon)+\delta v(2))+(1-\gamma) \delta v(3)]+\left(1-\beta_{m}\right) \delta v(0)\right\} .
\end{align*}
$$

For the expert's recommendation strategy to be optimal, it is necessary that

$$
\begin{equation*}
\delta v(0) \geq \gamma[(1-\delta)(p-c+\varepsilon)+\delta v(2)]+(1-\gamma) \delta v(3) \tag{29}
\end{equation*}
$$

and

$$
\begin{equation*}
\gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4) \geq \delta v(1) \tag{30}
\end{equation*}
$$

Condition (29) ensures that the expert weakly prefers no treatment for the minor problem and condition (30) says that the expert weakly prefers treatment for the serious problem.
i) We first characterize the upper bound of $V^{2}$. Using (29) and (30), it is without loss of generality to reduce (28) to

$$
\begin{equation*}
v=\alpha\{\gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4)\}+(1-\alpha) \delta v(0) \tag{31}
\end{equation*}
$$

The maximum value of $V^{2}$ is achieved by solving:

$$
\begin{equation*}
\underset{p, \beta, \gamma, v(.)}{\operatorname{Max}} \tag{31}
\end{equation*}
$$

Subject to (29), (30),

$$
\begin{gather*}
c \leq p  \tag{32}\\
\gamma=\left\{\begin{array}{cc}
c & \leq \beta_{m}<\beta_{s} \leq 1 \\
{[0,1]} & \text { for } p<E(l \mid \beta) \\
0 & p=E(l \mid \beta) \\
0 & p=E(l \mid \beta)
\end{array}\right. \tag{33}
\end{gather*}
$$

Condition (34) ensures that $\gamma$ is the client's best response.

We first show that it is optimal to choose $v(1)=v(3)=0$ and $v(0)=v(4)=\bar{v}$. The former holds because (31) does not involve $v(1)$ or $v(3)$ and constraints (29) and (30) are relaxed at $v(1)=v(3)=0$.

Next, we show $v(0)=v(4)=\bar{v}$. Note that given $p \geq c$, after substituting $v(1)=0$, constraint (30) is satisfied for all $v(2), v(4) \in[0, \bar{v}]$ and can be ignored. An increase in $v(0)$ and $v(4)$ weakly relaxes constraint (29) but increases (31). So, it is optimal to choose $v(0)=v(4)=\bar{v}$.

Next, we show Max $V^{2}=\delta \bar{v}\left[1+\frac{\alpha(1-\delta)\left(l_{s}-c\right)}{\delta \bar{v}+(1-\delta)\left(l_{s}-c+\varepsilon\right)}\right]$. Substitute $v(0)=v(4)=\bar{v}$ and $v(1)=$ $v(3)=0$ into (31) as well as (29) and ignore constraint (30), the expert's expected payoff maximization problem boils down to

$$
\begin{equation*}
\operatorname{Max}_{p, \gamma, \beta, v(2)} \alpha \gamma[(1-\delta)(p-c)+\delta(v(2)-\bar{v})]+\delta \bar{v} \tag{35}
\end{equation*}
$$

subject to $(32),(33),(34)$ and

$$
\begin{equation*}
\delta \bar{v} \geq \gamma[(1-\delta)(p-c+\varepsilon)+\delta v(2)] \tag{36}
\end{equation*}
$$

Following (36), $v(2) \leq \frac{\bar{v}}{\gamma}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}$. So, $v(2) \leq \min \left\{\frac{\bar{v}}{\gamma}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}, \bar{v}\right\}$. Let $\widehat{\gamma}=$ $\frac{\delta \bar{v}}{\delta \bar{v}+(1-\delta)(p-c+\varepsilon)}$. It can be verified that

$$
\min \left\{\frac{\bar{v}}{\gamma}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}, \bar{v}\right\}=\left\{\begin{array}{cl}
\bar{v} & \text { for } \gamma \leq \widehat{\gamma}  \tag{37}\\
\frac{\bar{v}}{\gamma}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta} & \text { for } \gamma>\widehat{\gamma}
\end{array}\right.
$$

Next, we show the optimal price is $p=E(l \mid \beta)$. Consider $p=E(l \mid \beta)$. Since (35) is increasing in $\gamma$ and $v(2)$, for $\gamma \leq \widehat{\gamma},(35)$ is maximized by $v(2)=\bar{v}$ and $\gamma=\widehat{\gamma}$, and is

$$
\begin{equation*}
\delta \bar{v}\left[1+\frac{\alpha(1-\delta)(E(l \mid \beta)-c)}{\delta \bar{v}+(1-\delta)(E(l \mid \beta)-c+\varepsilon)}\right] . \tag{38}
\end{equation*}
$$

For $\gamma>\hat{\gamma}$, by (37), it is optimal to set $v(2)=\frac{\bar{v}}{\gamma}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}$. Substitute $v(2)$ into (35), the expert's expected payoff is reduced to

$$
\begin{equation*}
-\gamma[\alpha(1-\delta) \varepsilon+\alpha \delta \bar{v}]+\delta \bar{v}(1+\alpha) \tag{39}
\end{equation*}
$$

which is decreasing in $\gamma$. Hence, the maximum value of $v$ is achieved by $\gamma=\hat{\gamma}$, which yields the expert (38).

Next, consider $p>E(l \mid \beta)$. The client's best response is $\gamma=0$ which yields the expert $\delta \bar{v}<(38)$. Lastly, consider $p<E(l \mid \beta)$, the client's best response is $\gamma=1$. By (37), it is optimal to set $v(2)=$ $\bar{v}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}$. Substitute the optimal values of $\gamma$ and $v(2)$ into (35), the expert's payoff is $\delta \bar{v}-$ $\alpha(1-\delta) \varepsilon<(38)$.

Lastly, we show it is optimal to set $\beta_{m}=0$ and $\beta_{s}=1$. Note that (38) is strictly increasing in $E(l \mid \beta)$ which increases in $\beta_{s}$ and decreases in $\beta_{m}$. Thus, it is optimal to have $\beta_{s}=1$ and $\beta_{m}=0$ which yields
$E(l \mid \beta)=l_{s}$. As a result,

$$
\operatorname{Max} V^{2}=\delta \bar{v}\left(1+\frac{\alpha(1-\delta)\left(l_{s}-c\right)}{\delta \bar{v}+(1-\delta)\left(l_{s}-c+\varepsilon\right)}\right)
$$

ii) We characterize the lower bound of $V^{2}$ and show Min $V^{2}=0$. Consider $v(1)=v(2)=v(3)=v(4)=0$, $p>E(l \mid \beta)$ and $\gamma=0$. The expert's payoff is 0 at these values, constraints (29) and (30) are both satisfied, and $\gamma=0$ is the client's best response. In summary, $V^{2}=\left[0, \delta \bar{v}\left(1+\frac{\alpha(1-\delta)\left(l_{s}-c\right)}{\delta \bar{v}+(1-\delta)\left(l_{s}-c+\varepsilon\right)}\right)\right]$.

Step 3 For $[0, \bar{v}]$ to be a self-generating set, it requires that

$$
[0, \bar{v}] \subset \cup_{i=1,2} V^{i}=\left[0, \delta \bar{v}\left(1+\frac{\alpha(1-\delta)\left(l_{s}-c\right)}{\delta \bar{v}+(1-\delta)\left(l_{s}-c+\varepsilon\right)}\right)\right]
$$

The upper bound $\bar{v}$ is solved from

$$
\begin{aligned}
\bar{v} & =\delta \bar{v}\left(1+\frac{\alpha(1-\delta)\left(l_{s}-c\right)}{\delta \bar{v}+(1-\delta)\left(l_{s}-c+\varepsilon\right)}\right) \Rightarrow \\
\bar{v} & =\pi^{m}=\alpha\left(l_{s}-c\right)-\frac{1-\delta}{\delta}\left(l_{s}-c+\varepsilon\right) .
\end{aligned}
$$

The set $\left[0, \pi^{m}\right]$ is non empty if and only if $\delta \geq \underline{\delta}^{m}(\alpha)$. Hence, for $p \geq c$, the maximum equilibrium payoff accrued to the expert is $\pi^{m}$ for $\delta \geq \underline{\delta}^{m}(\alpha)$ and is 0 otherwise. Q.E.D.

Lemma 6 When $p \in[c-\varepsilon, c), \bar{v}=\pi^{o}$ for $\alpha \geq \widetilde{\alpha}$ and $\delta \geq \underline{\delta}^{o}(\alpha) ; \bar{v}=0$, otherwise.

We divide action profiles into three categories: i) $\Omega_{3} \equiv\left\{p,\left\{\beta \mid 0<\min \left\{\beta_{m}, \beta_{s}\right\} \leq 1\right\}, \gamma\right\}$, ii) $\Omega_{4} \equiv$ $\left\{p,\left\{\beta \mid 0=\beta_{m} \leq \beta_{s} \leq 1\right\}, \gamma\right\}$ and iii) $\Omega_{5} \equiv\left\{p,\left\{\beta \mid 0=\beta_{s} \leq \beta_{m} \leq 1\right\}, \gamma\right\}$. Let $V^{i}, i=3,4,5$, denote the set of payoffs decomposed by action profile $\Omega_{i}$ on $[0, \bar{v}]$. The proof is divided into 4 steps. The first three steps characterize the sets of payoffs decomposed by action profiles $\Omega_{i}, i=3,4,5$, respectively. Step 4 characterizes the maximum self-generating set.

Step 1. Consider action profile $\Omega_{3}$. For the expert's recommendation strategy to be optimal, it is necessary that

$$
\begin{gather*}
\delta v(0) \leq \gamma[(1-\delta)(p-c+\varepsilon)+\delta v(2)]+(1-\gamma) \delta v(3), \text { and }  \tag{40}\\
\delta v(1) \leq \gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4) \tag{41}
\end{gather*}
$$

Constraints (40) and (41) ensure that the expert prefers treatment for both types of problem. Using (40) and (41), the expert's expected payoff can be written as

$$
\begin{align*}
v= & \alpha\{\gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4)\} \\
& +(1-\alpha)\{\gamma[(1-\delta)(p-c+\varepsilon)+\delta v(2)]+(1-\gamma) \delta v(3)\} \tag{42}
\end{align*}
$$

i) We characterize $\max V^{3}$ by solving the following problem:

$$
\max _{p, \beta, \gamma, v(.),}(42)
$$

subject to (40), (41)

$$
\begin{align*}
p & \in[c-\varepsilon, c)  \tag{43}\\
0 & <\min \left\{\beta_{m}, \beta_{s}\right\} \leq 1  \tag{44}\\
\gamma & =\left\{\begin{array}{cc}
{[0,1]} & p=E(l \mid \beta) \\
0 & p>E(l \mid \beta) \\
1 & p<E(l \mid \beta)
\end{array}\right. \tag{45}
\end{align*}
$$

We first show that it is optimal to set $v(0)=v(1)=0$ and $v(2)=v(3)=v(4)=\bar{v}$. Since $v(0)$ and $v(1)$ are not in (42) and reducing their values relaxes incentive constraints (40) and (41), it is optimal to choose $v(0)=v(1)=0$. Furthermore, because (42) increases in $v(2), v(3)$ and $v(4)$ and the incentive constraints are relaxed when these values are increased, it is optimal to set $v(2)=v(3)=v(4)=\bar{v}$. Substitute the optimal value of $v($.$) into (42) and constraints (40) and (41), the expert's payoff maximization problem is reduced to$

$$
\begin{equation*}
\operatorname{Max}_{p, \beta, \gamma} \gamma(1-\delta)\{p-c+(1-\alpha) \varepsilon\}+\delta \bar{v} \tag{46}
\end{equation*}
$$

Subject to (43), (44), (45), and

$$
\begin{gather*}
0 \leq \gamma(1-\delta)(p-c+\varepsilon)+\delta \bar{v}  \tag{47}\\
0 \leq \gamma(1-\delta)(p-c)+\delta \bar{v} \tag{48}
\end{gather*}
$$

, where (47) follows from (40) and (48) follows from (41).

Next, we show

$$
\max V^{3}=\left\{\begin{array}{cl}
\max \left\{\frac{\delta \bar{v}(1-\alpha) \varepsilon}{(c-E(l))}, \delta \bar{v}\right\} & \text { if } \bar{v}<\frac{(1-\delta)(c-E(l))}{\delta}  \tag{49}\\
\max \{(1-\delta)\{E(l)-c+(1-\alpha) \varepsilon\}+\delta \bar{v}, \delta \bar{v}\} & \text { if } \bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}
\end{array}\right.
$$

First note that the solution for the constrained maximization problem requires $\gamma>0$ Suppose $\gamma=0$, then $(46)=\delta \bar{v}$. But an increase in $\gamma$ increases (46) without violating the constraints. This constitutes a contradiction. For $\gamma>0$ to be the consumer's best response, it is necessary that $p \leq E(l \mid \beta)$. Since (46) increases in $p$ and constraints (47) and (48) are relaxed when $p$ increases, it is optimal to have $p=E(l \mid \beta)$. Furthermore, constraint (48) implies (47) holds with strict inequality. Thus, $\beta_{m}=1$. Given $\beta_{m}=1, E(l \mid \beta)$ increases in $\beta_{s}$ and reaches the maximum value $E(l)$ at $\beta_{s}=1$. Substitute $p=E(l \mid \beta=1)$ into (48), it follows that $\gamma \leq \frac{\delta \bar{v}}{(1-\delta)(c-E(l))}$. So, $\gamma \leq \bar{\gamma} \equiv \min \left\{1, \frac{\delta \bar{v}}{(1-\delta)(c-E(l))}\right\}$. It can be verified that

$$
\bar{\gamma}=\left\{\begin{array}{cl}
\frac{\delta \bar{v}}{\frac{(1-\delta)(c-E(l))}{}} & \text { if } \bar{v}<\frac{(1-\delta)(c-E(l))}{\delta}  \tag{50}\\
1 & \text { if } \bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}
\end{array}\right.
$$

Because (46) increases in $\gamma$, the expert's payoff is maximized at $\bar{\gamma}$ and is

$$
\left\{\begin{array}{cl}
\frac{\delta \bar{v}(1-\alpha) \varepsilon}{(c-E(l))} & \text { if } \bar{v}<\frac{(1-\delta)(c-E(l))}{\delta} \\
(1-\delta)\{E(l)-c+(1-\alpha) \varepsilon\}+\delta \bar{v} & \text { if } \bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}
\end{array}\right.
$$

Next, we derive the lower bound of $V^{3}$. It can be verified that min $V^{3}=0$, which is obtained by $v(0)=v(1)=v(2)=v(3)=v(4)=0, \gamma=0$ and $p>E(l \mid \beta)$. In summary,

$$
V^{3}=\left(\begin{array}{cl}
{\left[0, \max \left\{\frac{\delta \bar{v}(1-\alpha) \varepsilon}{(c-E(l))}, \delta \bar{v}\right\}\right]} & \text { if } \bar{v}<\frac{(1-\delta)(c-E(l))}{\delta}  \tag{51}\\
{[0, \max \{(1-\delta)\{E(l)-c+(1-\alpha) \varepsilon\}+\delta \bar{v}, \delta \bar{v}\}]} & \text { if } \bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}
\end{array} .\right.
$$

Step 2 Consider action profile $\Omega_{4}$. Given $p<c$ and $\beta_{m}=0$, the consumer's best response is $\gamma=1$. Substitute $\gamma=1$ and $\beta_{m}=0$ into (28), the expert's payoff is reduced to

$$
\begin{equation*}
\alpha\left\{\beta_{s}[(1-\delta)(p-c)+\delta v(2)]+\left(1-\beta_{s}\right) \delta v(1)\right\}+(1-\alpha) \delta v(0) \tag{52}
\end{equation*}
$$

We first show Max $V^{4}=\delta \bar{v}$. The maximum value of $V^{4}$ is obtained by the following maximization program:

$$
\begin{equation*}
\max _{v(0), v(2), p, \beta_{s}, \gamma}(52) \tag{53}
\end{equation*}
$$

subject to (43), (45)

$$
\begin{gather*}
0 \leq \beta_{s} \leq 1  \tag{54}\\
\delta v(0) \geq[(1-\delta)(p-c+\varepsilon)+\delta v(2)] \tag{55}
\end{gather*}
$$

where (55) ensures $\beta_{m}=0$.

We show it is optimal to have $v(0)=v(1)=\bar{v}$. To see this, an increase in $v(0)$ or $v(1)$ weakly relaxes (55) and strictly increases the expert's expected payoff, so $v(0)=v(1)=\bar{v}$. Constraint (55) implies $v(2) \leq$ $\bar{v}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}$. Since (53) increases in $v(2)$, it is optimal to set $v(2)=\bar{v}-\frac{(1-\delta)(p-c+\varepsilon)}{\delta}$. Substitute the optimal values of $v(0), v(1), v(2)$ into (53), the expert's expected payoff boils down to

$$
v=\alpha\left\{\delta \bar{v}-\beta_{s}(1-\delta) \varepsilon\right\}+(1-\alpha) \delta \bar{v}
$$

which is maximized at $\beta_{s}=0$ and hence $\operatorname{Max} V^{4}=\delta \bar{v}$.

Next, we show $\min V^{4}=0$. The minimum value of $V^{4}$ is solved by minimizing (52) subject to (43), (45), (54) and (55). It can be verified that when $\beta_{s}=0, v(1)=v(2)=0, p=c-\varepsilon$ and $v(0)=\frac{(1-\delta)(p-c+\varepsilon)}{\delta}=$ 0 , the expert's payoff is 0 . In summary $V^{4}=[0, \delta \bar{v}]$.

Step 3 Consider action profile $\Omega_{5}$. Given $0=\beta_{s} \leq \beta_{m}$ and $l_{m}<c-\varepsilon \leq p$, the client's best response is $\gamma=0$. For the expert's recommendation strategy to be optimal, it is necessary that

$$
\begin{equation*}
\gamma[(1-\delta)(p-c)+\delta v(2)]+(1-\gamma) \delta v(4) \leq \delta v(1) \tag{56}
\end{equation*}
$$

Substitute $\beta_{s}=0, \gamma=0$ into (28), it is reduced to

$$
\begin{equation*}
\alpha \delta v(1)+(1-\alpha) \delta\left\{\beta_{m} v(3)+\left(1-\beta_{m}\right) v(0)\right\} . \tag{57}
\end{equation*}
$$

We choose $\beta_{m}, p$ and $v($.$) to maximize (57) subject to \beta_{m} \in[0,1],(43),(45)$, and

$$
\begin{equation*}
v(4) \leq v(1) \tag{58}
\end{equation*}
$$

where (58) follows from (56) after substituting $\beta_{s}=0$ and $\gamma=0$. It is optimal to set $v(1)=v(3)=v(0)=\bar{v}$ which yields $\max V^{5}=\delta \bar{v}$. It can be verified that min $V^{5}=0$ which is obtained by $v(0)=v(1)=v(2)=$ $v(3)=v(4)=0, p>E(l \mid p)$ and $\gamma=0$. Hence, $V^{5}=[0, \delta \bar{v}]$

Step 4 We characterize the maximum self-generating set. The set $[0, \bar{v}]$ is self-generating if and only if

$$
[0, \bar{v}] \subset \cup_{i=3,4,5} V^{i}
$$

Suppose $\bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}$. The union

$$
\cup_{i=3,4,5} V^{i}=[0, \max \{(1-\delta)(E(l)-c+(1-\alpha) \varepsilon)+\delta \bar{v}, \delta \bar{v}\}]
$$

Let $\widetilde{\alpha}=\frac{c-\varepsilon-l_{m}}{l_{s}-\varepsilon-l_{m}}$. Then,

$$
\cup_{i=3,4,5} V^{i}=\left\{\begin{array}{cc}
{[0,(1-\delta)(E(l)-c+(1-\alpha) \varepsilon)+\delta \bar{v}]} & \text { for } \alpha>\widetilde{\alpha} \\
{[0, \delta \bar{v}]} & \text { for } \alpha \leq \widetilde{\alpha}
\end{array}\right.
$$

So, when $\alpha>\widetilde{\alpha}, \bar{v}$ is solved from

$$
\begin{aligned}
\bar{v} & =(1-\delta)\{E(l)-c+(1-\alpha) \varepsilon\}+\delta \bar{v} \Leftrightarrow \\
\bar{v} & =E(l)-c+(1-\alpha) \varepsilon
\end{aligned}
$$

The maximum value is sustainable if and only if

$$
\begin{aligned}
E(l)-c+(1-\alpha) \varepsilon & \geq \frac{(1-\delta)(c-E(l))}{\delta} \\
\delta & \geq \underline{\delta}^{o}(\alpha)
\end{aligned}
$$

When $\alpha \leq \widetilde{\alpha}, \bar{v}$ is solved from $\bar{v}=\delta \bar{v}$, and the only solution is $\bar{v}=0$.

$$
\text { Now, consider } \bar{v}<\frac{(1-\delta)(c-E(l))}{\delta}, \cup_{i=3,4,5} V^{i}=\left[0, \max \left\{\frac{\delta \bar{v}(1-\alpha) \varepsilon}{(c-E(l))}, \delta \bar{v}\right\}\right] \text {. The solution for the }
$$ maximum value is solved from

$$
\begin{equation*}
\bar{v}=\bar{v} \max \left\{\frac{\delta(1-\alpha) \varepsilon}{(c-E(l))}, \delta\right\} \tag{59}
\end{equation*}
$$

When $\alpha>\widetilde{\alpha}$ and $\delta=\delta^{o}, \max \left\{\frac{\delta(1-\alpha) \varepsilon}{(c-E(l))}, \delta\right\}=1$ holds, which implies (59) is satisfied for all $\bar{v}$. Thus, in this case, $\bar{v}$ is bounded above by $\frac{\left(1-\underline{\delta}^{o}(\alpha)\right)(c-E(l))}{\underline{\delta}^{o}(\alpha)}=E(l)-c+(1-\alpha) \varepsilon$. When $\alpha \leq \widetilde{\alpha}$ or $\delta \neq \underline{\delta}^{o}(\alpha)$, $\max \left\{\frac{\delta(1-\alpha) \varepsilon}{(c-E(l))}, \delta\right\}<1$. The only solution for (59) is $\bar{v}=0$.

So, taking into account of the case $\bar{v} \geq \frac{(1-\delta)(c-E(l))}{\delta}$ and the case $\bar{v}<\frac{(1-\delta)(c-E(l))}{\delta}$, we conclude that $\bar{v}=\pi^{o}$ for $\alpha>\widetilde{\alpha}$ and $\delta \geq \underline{\delta}^{o}(\alpha)$ and 0 otherwise. Q.E.D.

## References

[1] Abreu, D., D. Pearce, and E. Stacchetti (1990), "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," Econometrica, 58, 1041-1063.
[2] Alger, I. and F. Salanie (2006), "A Theory of Fraud and Over-treatment in Experts Markets," Journal of Economics and Management Strategy, 15, 853-881.
[3] Atreja, A., N. Bellam, and S.R. Levy (2005), "Strategies to enhance patient adherence: making it simple," Medscape General Medicine, 7, 4.
[4] Bar-Isaac, H. and S. Tadelis (2008), "Seller reputation," Foundations and Trends in Microeconomics, 4, 273-351.
[5] Bester, H., and M. Dahm (2017), "Credence Goods, Costly Diagnosis and Subjective Evaluation," The Economic Journal, forthcoming.
[6] Chen, Y., J. Li and J. Zhang (2017), "Liability in Markets for Credence Goods," working paper.
[7] Darby, M. and E. Karni (1973), "Free Competition and the Optimal Amount of Fraud," Journal of Law and Economics 16, 67-88.
[8] Dulleck, U. and R. Kerschbamer (2006), "On Doctors, Mechanics, and Computer Specialists: The Economics of Credence Goods," Journal of Economic Literature, 44, 5-42.
[9] Dulleck, U. and R. Kerschbamer (2009), "Experts vs. Discounters: Consumers Free-Riding and Experts Withholding Advice in Markets for Cre- dence Goods," International Journal of Industrial Organization, 27, 15-23.
[10] Dulleck, U., R. Kerschbamer, and M. Sutter (2011), "The Economics of Credence Goods: an Experiment on The Role of Liability, Verifiability, Reputation, and Competition," American Economic Review, 101, 526-555.
[11] Ely, J. and J. Välimäki (2003), "Bad Reputation," Quarterly Journal of Economics 118, 785-814.
[12] Emons, W. (1997), "Credence Goods and Fraudulent Experts," The RAND Journal of Economics, 28, 107-119.
[13] Emons, W. (2001), "Credence Goods Monopolists," International Journal of Industrial Organization, 19, 375-389.
[14] Fong, Y. (2005), "When do Experts Cheat and Whom do They Target," The RAND Journal of Economics, 36, 113-130.
[15] Fong, Y., T. Liu, and D. Wright (2014), "On the Role of Verifiability and Commitment in Credence Goods Markets," International Journal of Industrial Organization, 37, 118-129
[16] Frankel, A. and M. Schwarz (2014), "Experts and Their Records," Economic Inquiry, 52, 56-71
[17] Hafner, S. and C. Taylor (2017), "Advice is Cheap: Information is Not!" working paper
[18] Hörner, J. (2002) "Reputation and Competition," American Economic Review, 92, 644-663
[19] Liu, T. (2011), "Credence Goods Markets with Conscientious and Selfish Experts," International Economic Review, 52, 227-244.
[20] Liu, T., E. Vergara-Cobos, and Y. Zhou (2017), "Pricing Schemes and Seller Fraud: Evidence from New York City Taxi Rides," working paper.
[21] Klein, B. and K. Leffler (1981), "The Role of Market Forces in Assuring Contractual Performance," Jounal of Political Economy, 89, 615-641
[22] Mailath, G. and L. Samuelson (2006), "Repeated Games and Reputations," Oxford University Press.
[23] Mimra, W., A. Rasch, and C. Waibel (2016), "Price Competition and Reputation in Credence Goods Markets: Experimental Evidence," Games and Economic Behavior, 100, 337-352.
[24] Park, I. (2005), "Cheap Talk Referrals of Differentiated Experts in Repeated Relationship," The RAND Journal of Economics, 36, 391-411 (2005).
[25] Pesendorfer, W. and A. Wolinsky (2003), "Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice," the Review of Economic Studies, 70, 417-437.
[26] Piette, J.D., M. Heisler, S. Krein, and E.A. Kerr (2005), "The role of patient-physician trust in moderating medication nonadherence due to cost pressures," Archives of Internal Medicine, 165, 1749-1755.
[27] Pitchik, C., and A. Schotter (1987), "Honesty in a Model of Strategic Information Transmission," American Economic Review, 77, 1032-36.
[28] Sihvonen, R. et. al. (2013), "Arthroscopic Partial Meniscectomy versus Sham Surgery for a Degenerative Meniscal Tear," 369, 2515-2524.
[29] Schneider, H. (2012), "Agency Problems and Reputation in Expert Services: Evidence from Auto Repair," Journal of Industrial Economics, 60, 406-433.
[30] Stahl, D. (1989), "Oligopolistic Pricing with Sequential Consumer Search," American Economic Review, 79, 700-712.
[31] Taylor, C.R. (1995)"The Economics of Breakdowns, Checkups, and Cures," Journal of Political Economy, 103, 53-74.
[32] Wolinsky, A. (1993), "Competition in a Market for Informed Experts' Services," The RAND Journal of Economics, 24, 380-398.
[33] Wolinsky, A. (1995), "Competition in markets for credence goods," Journal of Institutional and Theoretical Economics, 151, 117-131.


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[^1]:    ${ }^{1}$ "Patients give horror stories as cancer doctor gets in prison", CNN, July 11, 2015.

[^2]:    ${ }^{2}$ For an excellent review of the voluminous literature on reputation and trust of experience goods sellers, please see Bar-Isaac and Tadelis (2008).

[^3]:    ${ }^{3}$ This assumption can be justified by the increasing popularity of websites like Angie's list, Yelp, and RateMDs on which consumers actively post and share reviews on experts' services.
    ${ }^{4}$ This assumption is not crucial to our main findings in the repeated-game setting. This issue will be discussed in Section 6.
    ${ }^{5}$ In our main model, expert service is modeled as a credence good. In Section 3, we formally specify an alternative model in which the expert service is an experience good. By then, the distinction between credence expert service and experience expert service will be clear.
    ${ }^{6}$ We formally demonstrate this result in a benchmark model of experience services markets in Section 4.1.

[^4]:    ${ }^{7}$ A study by Sihvonen R. et al. (2013) found that for many patients, arthroscopic surgery is no more effective at relieving symptoms than rest, exercise and over-the-counter pain killers. The study is reported in the Atlantic and CBS news. See "When Evidence Says No, but Doctors Say Yes," and "Common arthroscopic knee surgery no better than "sham" version, researchers say".

[^5]:    ${ }^{8}$ In many real-life situations, the cost of a treatment depends on the complexity of a client's problem. For example, a cardiac surgeon spends less time on a by pass surgery when a patient's problem is mild than when her condition is serious. Similarly, it requires a tax lawyer less effort to file tax return for a taxpayer with a relatively simple tax situation.
    ${ }^{9}$ Pitchik and Schotter (1987), Wolinsky (1993), Fong (2005) and Liu (2011), Dulleck and Kerschbamber (2006).
    ${ }^{10}$ Our results continue to hold qualitatively if we relax the liability assumption. When the expert is not liable for fixing the consumer's problem, there is still no trade in the static Nash equilibrium. In the repeated game, if the expert charges a consumer for treatment which is not provided, the consumer will suffer the loss at the end of the period. Thus, the expert's fraudulent behavior will be perfectly revealed to future consumers and trigger the punishment phase. To support the monitoring-by-rejection or the one-price-fix-all equilibrium, which we define in Section 4.2 , we just need to add incentive constraints to ensure the expert does not take the money and run. These conditions are satisifed when the discount factor is high enough.

[^6]:    ${ }^{11}$ The use of public randomization device is common in the repeated game literature. This allows the expert and subsequent clients to publicly randomize at the beginning of the next period and hence will convexify the equilibrium payoff set.

[^7]:    ${ }^{12}$ To see another difference, we can verify that in our setting consumers' acceptance rate for treatments increases in $l_{s}\left(\right.$ i.e., $\left.\partial \gamma^{*} / \partial l_{s}>0\right)$. However, in Fong, the acceptance rate for the minor treatment stays at one when $l_{s}$ increases but the acceptance rate for the expensive treatment decreases in $l_{s}$. So the overall acceptance decreases in $l_{s}$.

[^8]:    ${ }^{13}$ Search cost for the first visit is sunk when the consumer decides whether to seek second opinion and when experts decide their recommendation policies. So it has no impact on these decisions. Nevertheless, a positive search cost for the first visit will reduce the consumer's maximimum willingness to pay for the first expert's treatment, so consumers' participation constraint (4) should be modified. Nevertheless, since the participation constraint is not binding in equilibrium, the introduction of search cost for the first visit does not change the equilibrium characterized in Proposition 5.

[^9]:    ${ }^{14}$ To give an example, Wolinsky (1995) argued that "the basic force that imitages experts' incentive to misrepresent minor treatment as major ones is customers' search for multiple opinions."

